Sequences and Proofs – Section Test (Answers)

1)

Each term decreases by 3, so the nth term must involve $-3n$

So $a = -3$

2)

nth term is $-3n + b$

1st term is 20, so $-3 + b = 20$

$b = 23$

3)

The sequence has nth term $pn^2 + qn + r$.

Terms	2	9	18	29	42
Differences		7	9	11	13
Second differences			2	2	2

So $p = 1$.

4)

Terms	2	9	18	29	42
pn^2	1	4	9	16	25
$qn + r$	1	5	9	13	17

The values of $qn + r$ go up by 4 each time, so $q = 4$.

5)

The nth term is $n^2 + 4n + r$

1st term = 2, so $1 + 4 + r = 2$

so $r = -3$

6)

nth term $= n(n+1)$

5th term $= 5(5+1) = 5 \times 6 = 30$

7)

nth term $= n(n+1)$

$240 = n(n+1)$

$n^2 + n - 240 = 0$

$(n-15)(n+16) = 0$

$n = 15$ or $n = -16$

Since the number of terms must be positive, there are 15 terms in the sequence.

8)

$5^{th}\text{ term} = 5^2 + 5k - 3 = 22 + 5k$

$9^{th}\text{ term} = 9^2 + 9k - 3 = 78 + 9k$

$78 + 9k = 3(22 + 5k)$

$78 + 9k = 66 + 15k$

$12 = 6k$

$k = 2$

9)

$$\text{nth term} = \frac{3 - 2n}{8n + 1}$$

$$3^{rd}\text{ term} = \frac{3 - 2\times 3}{8\times 3 + 1} = \frac{3 - 6}{25 + 1} = \frac{-3}{25} = -\frac{3}{25}$$

10)

As $n \to \infty$, $3 - 2n \to -2n$

$8n + 1 \to 8n$

$$\frac{3 - 2n}{8n + 1} \to \frac{-2n}{8n} = -\frac{1}{4}$$

The limit of the sequence is -0.25.