Worked Solutions

Centre Number		Candidate Number	
Surname	MR BARTON		
Other Names			
Candidate Signature			

For Examiner's Use	
Examiner's Initials	
Pages	Mark
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TOTAL	

AQA

Level 2 Certificate in Further Mathematics
June 2013

Further Mathematics 8360/2

Level 2

Paper 2 Calculator

Friday 21 June 2013 9.00 am to 11.00 am

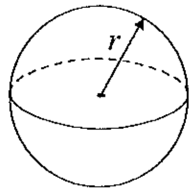
For this paper you must have:
- a calculator
- mathematical instruments.

Time allowed
- 2 hours

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.
- If your calculator does not have a π button, take the value of π to be 3.14 unless another value is given in the question.

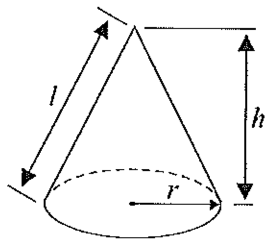
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

Formulae Sheet



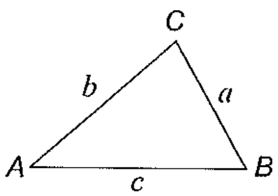
Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Volume of cone $= \frac{1}{3}\pi r^2 h$

Curved surface area of cone $= \pi r l$



In any triangle *ABC*

Area of triangle $= \frac{1}{2}ab\sin C$

Sine rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Quadratic Equation

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

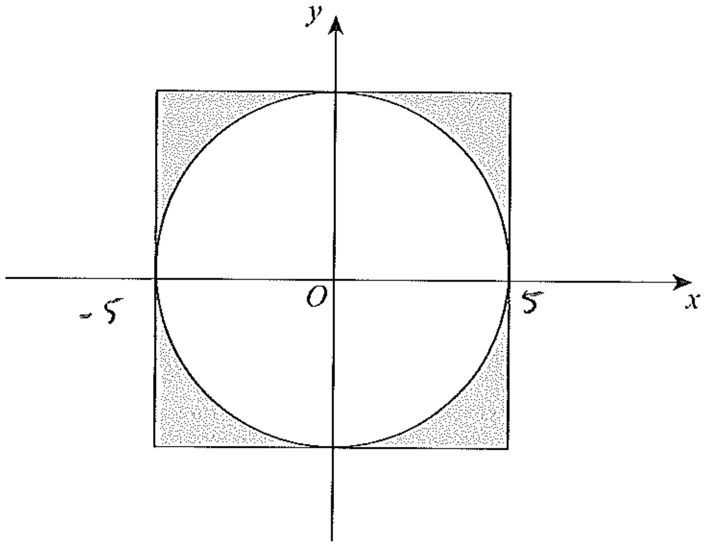
Trigonometric Identities

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\sin^2\theta + \cos^2\theta \equiv 1$

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The circle $x^2 + y^2 = 25$ touches each side of the square as shown.



Not drawn accurately

Work out the total shaded area.

radius = $\sqrt{25}$ = 5

Area of square = 10 × 10 = 100 cm²

Area of circle = $\pi r^2 = \pi \times 5^2 = 25\pi$

Shaded area = $100 - 25\pi$

= 21.4601...

Answer 21.46 cm² *(3 marks)*

3

Turn over ▶

Do not write outside the box

2 w is an integer such that $6 \leqslant 3w < 18$
x is an integer such that $-4 \leqslant x \leqslant 3$

2 (a) Work out **all** the possible integer values of w.

$6 \leq 3w < 18$

$\div 3 \{ \; 2 \leq w < 6$

Answer $2, 3, 4, 5$ *(3 marks)*

2 (b) Write down the **highest** possible value of x^2

Answer $(-4)^2 = 16$ *(1 mark)*

2 (c) Work out the **lowest** possible value of $w - x$

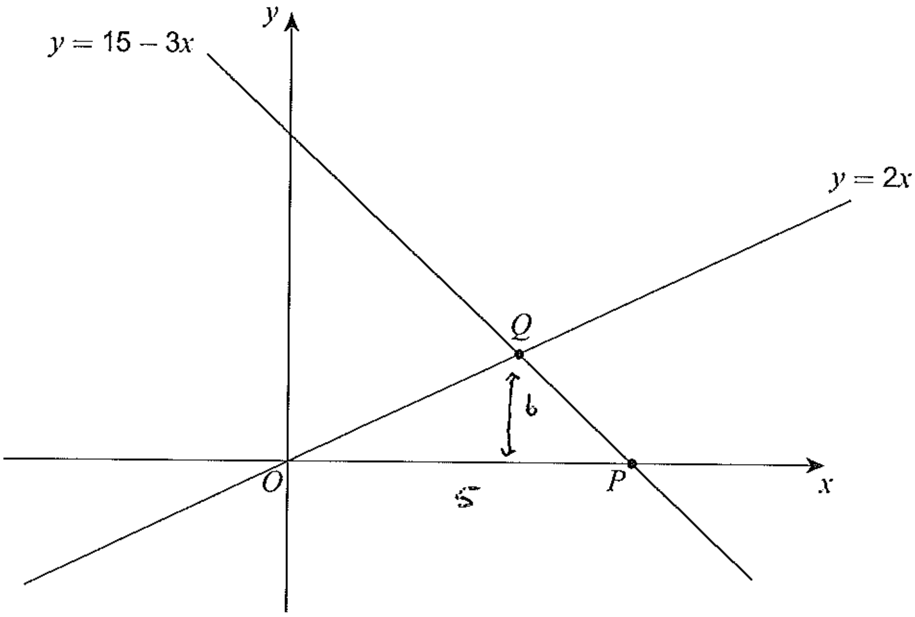
If $w = 2$, $x = 3$

$\rightarrow 2 - 3 = -1$

Answer -1 *(2 marks)*

Do not write outside the box

3 The sketch graphs of two straight lines are shown.



3 (a) Work out the coordinates of P.

y = 15 - 3x AND y = 0 → 15 - 3x = 0
15 = 3x
5 = x

Answer (5 , 0) *(1 mark)*

3 (b) Work out the coordinates of Q.

Lines cross: y = 15 - 3x AND y = 2x

→ 2x = 15 - 3x

+3x 5x = 15 y = 2(3)

÷5 x = 3 = 6

Answer (3 , 6) *(3 marks)*

3 (c) Use your answers to parts (a) and (b) to work out the area of triangle OPQ.

A = ½ × b × h = ½ × 5 × 6

= 15

Answer 15 *(2 marks)*

12

Turn over ▶

Do not write outside the box

4 You are given that $m : n = 2 : 5$

4 (a) Write m in terms of n.

$m : n$

$2 : 5$

$\frac{2}{5} : 1$

$m =$ $\frac{2}{5} n$ *(1 mark)*

4 (b) You are also given that $a : b = 10m : 3n$

Work out $a : b$ where a and b are integers.

$10m : 3n$

$10 (\frac{2}{5} n) : 3n$

$\frac{20}{5} n : 3n$

$4n : 3n$

Answer $4 : 3$ *(2 marks)*

Do not write outside the box

5 $y = (5x - 3)^2$

Work out $\dfrac{dy}{dx}$

Give your answer in the form $a(bx - c)$ where a, b and c are integers > 1

$y = (5x - 3)(5x - 3)$

$y = 25x^2 - 15x - 15x + 9$

$y = 25x^2 - 30x + 9$

$\dfrac{\partial y}{\partial x} = 50x - 30$

$= 10(5x - 3)$

$\dfrac{dy}{dx} =$ $10(5x - 3)$ *(4 marks)*

Turn over for the next question

7

Do not write outside the box

6 (a) Show that $\dfrac{c^2+5c+4}{3c+3}$ simplifies to $\dfrac{c+4}{3}$

$\div (c+1)$ $\Big\}$ $\dfrac{(c+4)(c+1)}{3(c+1)} = \dfrac{c+4}{3}$

(2 marks)

6 (b) Hence, or otherwise, simplify fully $\dfrac{c^2+5c+4}{3c+3} + \dfrac{3-2c}{6}$

using a)

$\rightarrow \dfrac{c+4}{3} + \dfrac{3-2c}{6}$

$\times 2$ $\quad \dfrac{2c+8}{6} + \dfrac{3-2c}{6}$

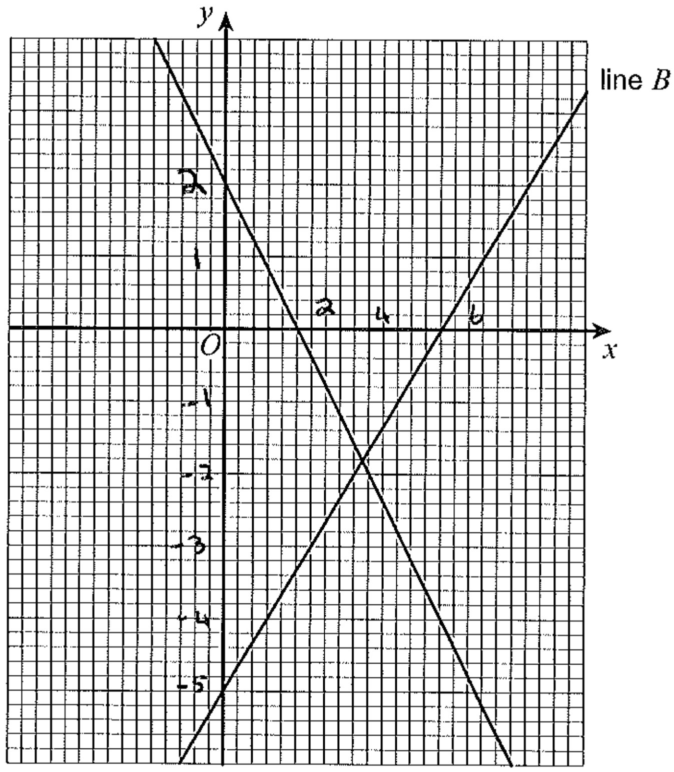
$\rightarrow \dfrac{2c+8+3-2c}{6}$

$\rightarrow \dfrac{11}{6}$

Answer $11/6$ .... *(3 marks)*

Do not write outside the box

7 The graph shows two straight lines.



The equation of line A is $y = 2 - x$

Work out the equation of line B.

(A) y-intercept = 2 , crosses x-axis when y = 0

→ x = 2

(B) Gradient = 5/6

y intercept = (0, -5)

Equation = y = 5/6 x - 5

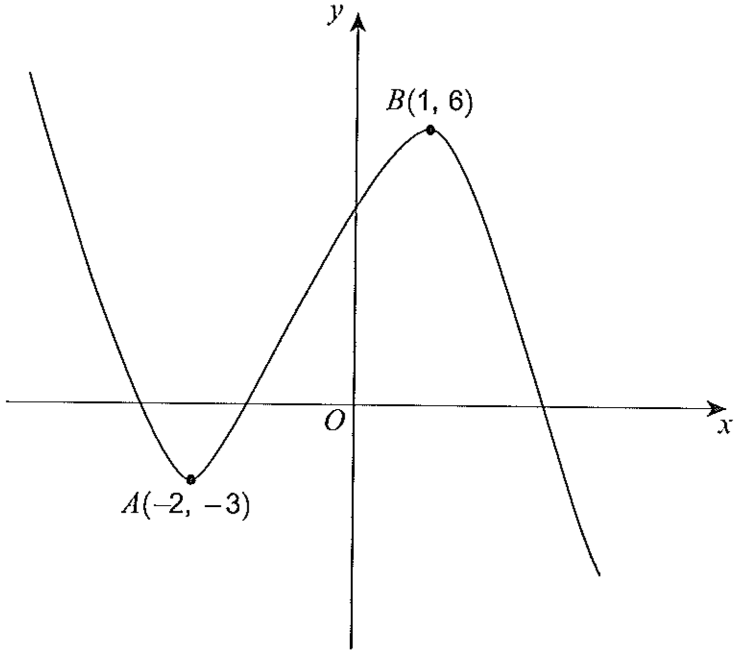
Answer .... (4 marks)

9

Turn over ▶

Do not write outside the box

8 A sketch of $y = f(x)$ is shown.
There are stationary points at A and B.



8 (a) Write down the equation of the tangent to the curve at A.

horizontal !

Answer $y = -3$ *(1 mark)*

8 (b) Write down the equation of the normal to the curve at B.

vertical !

Answer $x = 1$ *(1 mark)*

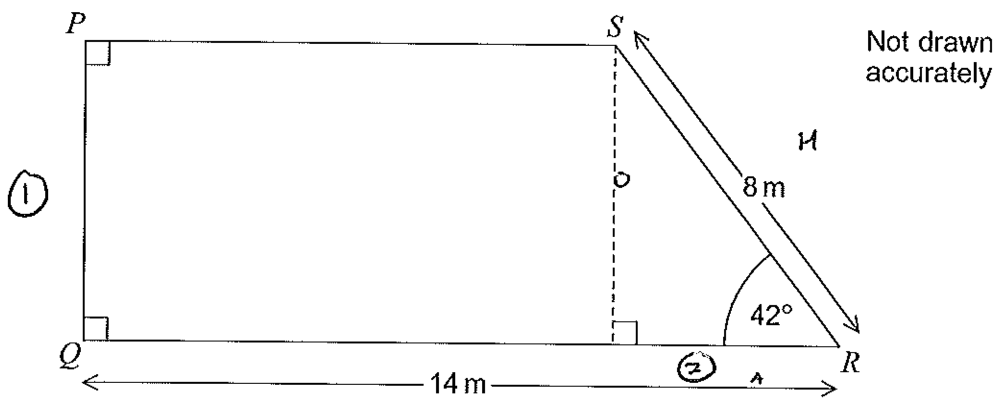
8 (c) Circle the range of values of x for which $f(x)$ is an increasing function.

$x < -2$ $\quad$ $-2 < x < 1$ $\quad$ $-3 < x < 6$ $\quad$ $x > 1$

(1 mark)

Do not write outside the box

9 $PQRS$ is a trapezium.



Work out the perimeter of $PQRS$.

① Need height: $\sin(42) = {}^{PQ}/_{8} \rightarrow PQ = 8 \times \sin(42)$

$= 5.3530$

② Need base: $\cos(42) = {}^{Base}/_{8} \rightarrow Base = 8 \times \cos(42)$

$= 5.94515$

$PS = 14 - Base = 8.0548$

Perimeter $= 14 + 8 + 8.0548 + 5.353$

$= 35.4$

Perimeter = 35.4 m *(5 marks)*

8

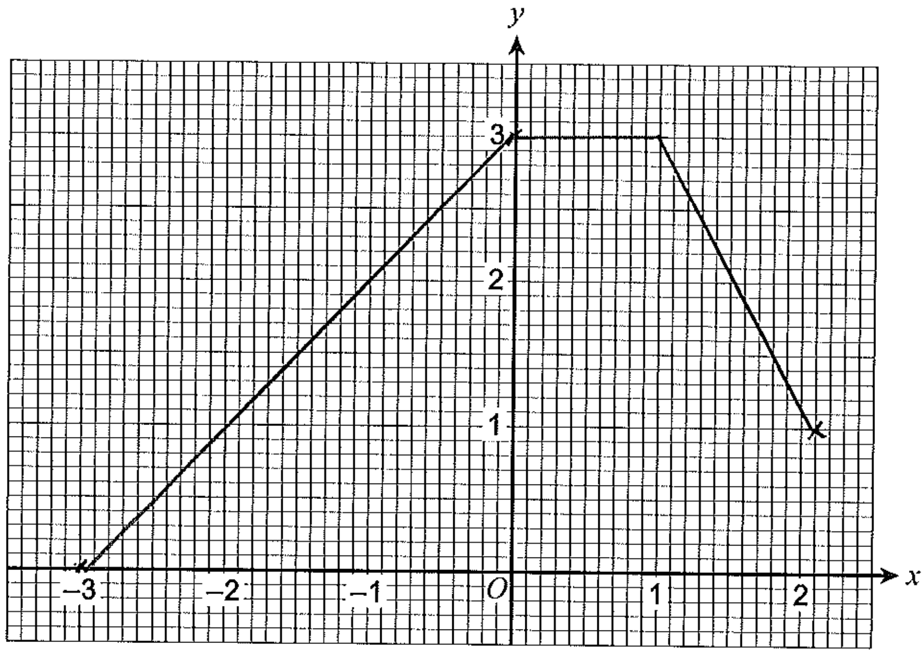
Turn over ▶

Do not write outside the box

10 A function f(x) is defined as

$$
\begin{aligned}
f(x) &= x + 3 \qquad & -3 \leqslant x < 0 \quad (1) \\
&= 3 & 0 \leqslant x < 1 \quad (2) \\
&= 5 - 2x & 1 \leqslant x \leqslant 2 \quad (3)
\end{aligned}
$$

Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 2$



(3 marks)

(1) $y = x + 3$

x	−3	−2	−1	0
y	0	1	2	3

(3) $y = 5 - 2x$

x	1	2
y	3	1

Do not write outside the box

11 (a) Work out $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}\begin{pmatrix} 0 & b \\ a & c \end{pmatrix}$

Give your answer in terms of a, b and c.

$$\begin{pmatrix} 0 & b \\ a & c \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ 1/3 & 0 \end{pmatrix}\begin{pmatrix} -a & 2b-c \\ 0 & 1/3 b \end{pmatrix}$$

① $(2 \times 0) + ((-1) \times a)$

② $(2 \times b) + ((-1) \times c)$

③ $(1/3 \times 0) + (0 \times a)$

④ $(1/3 \times b) + (0 \times c)$

Answer $\begin{bmatrix} -a & 2b-c \\ 0 & 1/3\, b \end{bmatrix}$ *(2 marks)*

11 (b) You are given that $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}\begin{pmatrix} 0 & b \\ a & c \end{pmatrix} = \mathbf{I}$ where **I** is the identity matrix.

Work out the values of a, b and c.

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

so $\begin{pmatrix} -a & 2b-c \\ 0 & 1/3\, b \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$-a = 1 \rightarrow a = -1$

$1/3\, b = 1 \rightarrow b = 3$

$2b - c = 0$

$2(3) - c = 0$

$6 - c = 0 \rightarrow c = 6$

$a =$ -1, $b =$ 3, $c =$ 6 *(3 marks)*

8

Turn over ▶

Do not write outside the box

12 Prove that $(5n+3)(n-1)+n(n+2)$
is a multiple of 3 for all integer values of n.

$5n^2 - 5n + 3n - 3 + n^2 + 2n$

$= 6n^2 - 3$

$= 3(2n^2 - 1)$

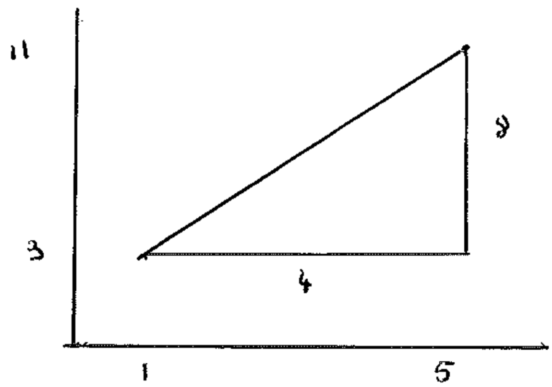
Anything multiplied by 3 is a multiple of 3

(4 marks)

13 The graph of $y = f(x)$ is a straight line.

The domain of $f(x)$ is $1 \leqslant x \leqslant 5$
The range of $f(x)$ is $3 \leqslant f(x) \leqslant 11$

Work out **one** possible expression for $f(x)$.



Gradient $= \frac{8}{4} = 2$

$x_1 = 1$

$y_1 = 3$

$m = 2$

$y - y_1 = m(x - x_1)$

$y - 3 = 2(x - 1)$

$y - 3 = 2x - 2$

$y = 2x + 1$

$f(x) =$ $2x + 1$ (4 marks)

Do not write outside the box

14 Work out an expression for the nth term of the quadratic sequence

11 15 21 29 39 .....

4 6 8 10

2 2 2

$\rightarrow n^2$

11 15 21 29 39

n^2 1 4 9 16 25

10 11 12 13 14

$1n + 9 = n + 9$

$\therefore n^2 + n + 9$

nth term = $n^2 + n + 9$ *(4 marks)*

12

Turn over ▶

Do not write outside the box

15 (a) $a^{11} \times b^6 \times c = a^9 \times b^{10}$

Write c in terms of a and b.
Give your answer in its simplest form.

$a^{11} \times b^6 \times c = a^9 \times b^{10}$

$\div a^{11} \times b^6$ $\quad c = \dfrac{a^9 \times b^{10}}{a^{11} \times b^6}$

$c =$ $a^{-2} \times b^4$ or b^4/a^2 (3 marks)

15 (b) $p^{-2} = q^6 \times r^4$

Write p in terms of q and r.
Give your answer in its simplest form.

$\dfrac{1}{p^2} = q^6 \times r^4$

$\times p^2$ $\quad 1 = p^2 \times q^6 \times r^4$

$\div (q^6 \times r^4)$ $\quad \dfrac{1}{q^6 \times r^4} = p^2$

$\sqrt{\ }$ $\quad \sqrt{\dfrac{1}{q^6 \times p^4}} = p \rightarrow p = (q^{-6} \times p^{-4})^{1/2}$

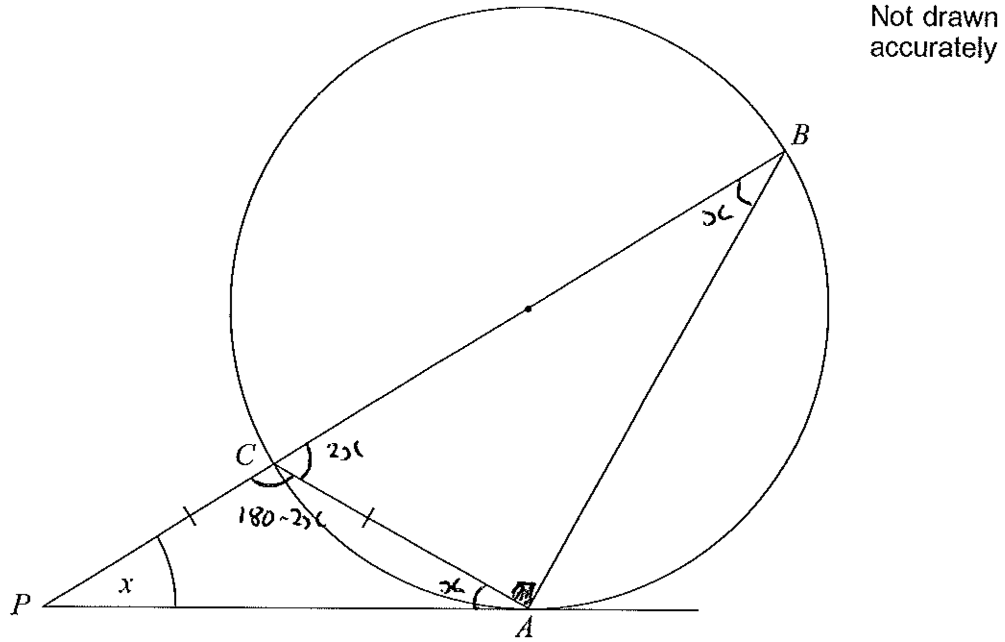
$= q^{-3} \times p^{-2}$

$p =$ .... (2 marks)

Do not write outside the box

16 A, B and C are points on the circumference of a circle.

- BC is a diameter
- BCP is a straight line
- AP is a tangent to the circle
- $PC = CA$



Work out the value of angle CPA, marked x on the diagram.

CAB = 90° (angles in semi-circle)

PAC = x (isosceles triangle)

PCA = 180 - 2x (angles in a triangle)

BCA = 2x (angles on straight line)

CBA = x (alternate segment theorem)

x + 2x + 90 = 180 (angles in a triangle)

→ 3x + 90 = 180

3x = 90 → x = 30

$x =$ 30 degrees *(5 marks)*

10

Turn over ▶

Do not write outside the box

17 Solve $\frac{4}{x-2}+\frac{1}{x+3}=5$

$$\frac{4(x+3)}{(x-2)(x+3)} + \frac{1(x-2)}{(x-2)(x+3)} = 5$$

$$\frac{4x+12+x-2}{1x^2+3x-2x-6} = 5$$

$$\frac{5x+10}{x^2+x-6} = 5$$

$\times(x^2+x-6)$ $\quad 5x+10 = 5(x^2+x-6)$

$5x+10 = 5x^2+5x-30$

$-5x$ $\quad 10 = 5x^2-30$

-10 $\quad 0 = 5x^2-40$

$\div 5$ $\quad 0 = x^2-8$

$+8$ $\quad 8 = x^2$

$\sqrt{\ }$ $\quad \pm\sqrt{8} = x$

Answer $x = \pm\sqrt{8}$ .... *(7 marks)*

Do not write outside the box

18 The curve $y = x^3 + bx + c$ has a stationary point at (–2, 20).

Work out the values of b and c.

$dy/dx = 3x^2 + b$

At stationary point, $dy/dx = 0$

$\rightarrow 3x^2 + b = 0$

we know when $x = -2$, $dy/dx = 0$

$\rightarrow 3(-2)^2 + b = 0$

$\rightarrow 12 + b = 0 \rightarrow b = -12$

when $x = -2$, $y = 20$

$\rightarrow y = (-2)^3 + (-12)(-2) + c = 20$

$\rightarrow -8 + 24 + c = 20$

$16 + c = 20$

$\rightarrow c = 4$

$b =$ -12

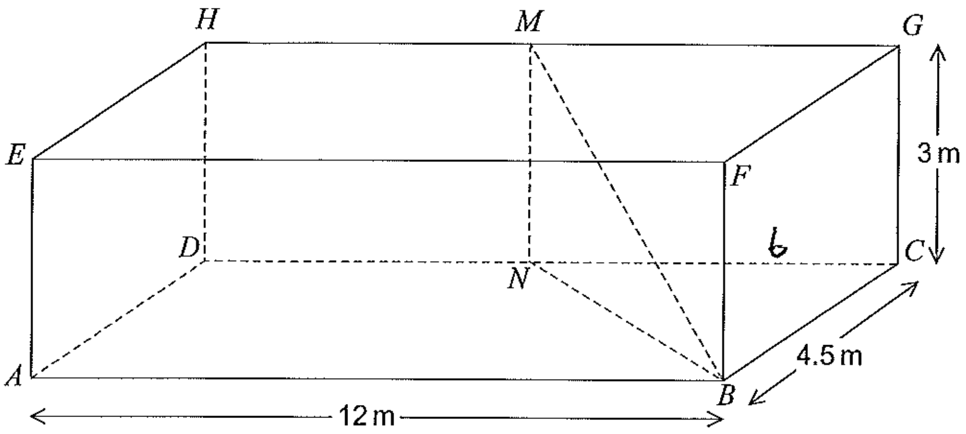
$c =$ 4 *(5 marks)*

Turn over for the next question

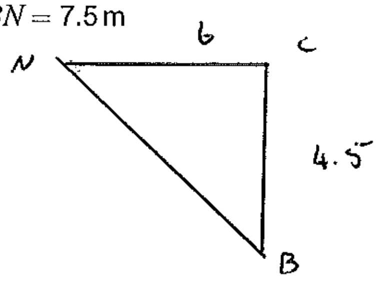
12

Do not write outside the box

19 $ABCDEFGH$ is a cuboid.
M is the midpoint of HG.
N is the midpoint of DC.



19 (a) Show that $BN = 7.5$ m



$BN = \sqrt{4.5^2 + 6^2}$

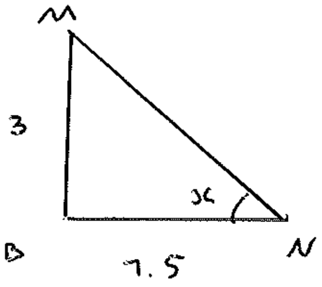
$= \sqrt{56.25}$

$= 7.5$

(2 marks)

Do not write outside the box

19 (b) Work out the angle between the line *MB* and the plane *ABCD*.



$\text{Tan}(x) = \frac{\text{Opp}}{\text{Adj}}$

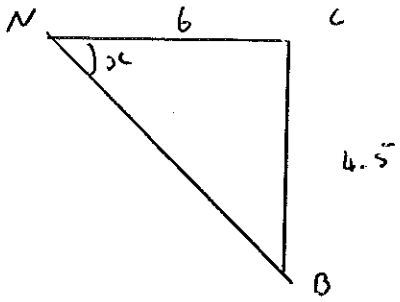
$\text{Tan}(x) = \frac{3}{7.5}$

$x = \tan^{-1}(3/7.5)$

$= 21.801...$

Answer 21.8 degrees *(2 marks)*

19 (c) Work out the **obtuse** angle between the planes *MNB* and *CDHG*.



$\text{Tan}(x) = \frac{4.5}{6}$

$x = \text{Tan}^{-1}\left(\frac{4.5}{6}\right)$

$= 36.869...$

Obtuse, so $180 - 36.869... = 143.13$

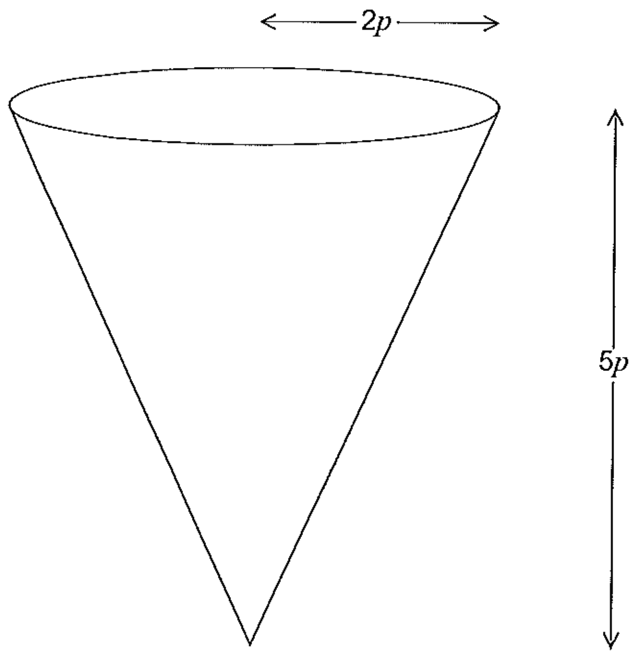
Answer 143 degrees *(2 marks)*

6

Turn over ▶

Do not write outside the box

20 This right circular cone has radius $2p$ and height $5p$.
The dimensions are in centimetres.



The volume of the cone is $22\,500\pi$ cm^3.

Work out the value of p.

Volume of a cone $= \frac{1}{3}\pi r^2 h = 22500\pi$

$= \frac{1}{3}\pi (2p)^2(5p) = 22\,500\pi$

$= \frac{1}{3}\pi (4p^2)(5p) = 22\,500\pi$

$= \frac{1}{3}\pi (20p^3) = 22\,500\pi$

$\times 3$ $\quad 20p^3\pi = 67\,500\pi$

$\div \pi$ $\quad 20p^3 = 67\,500$

$\div 20$ $\quad p^3 = 3375$

$\sqrt[3]{}$ $\quad p = \sqrt[3]{3375}$

$= 15$

$p =$ 15 cm *(4 marks)*

Do not write outside the box

21 $(x-a)$ is a factor of $2x^3 - 7ax + 3a$

Work out the **largest** possible value of a.

If $(x-a)$ is a factor, $f(a) = 0$

$\rightarrow 2(a)^3 - 7a(a) + 3a = 0$

$\rightarrow 2a^3 - 7a^2 + 3a = 0$

$\div a \left\{ 2a^2 - 7a + 3 = 0 \right.$

$(2a - 1)(a - 3) = 0$

$\downarrow \qquad \downarrow$

$a = 1/2 \qquad a = 3$

Answer $a = 3$ *(4 marks)*

22 Solve $\tan^2\theta + 3\tan\theta = 0$ for $0° < \theta < 360°$

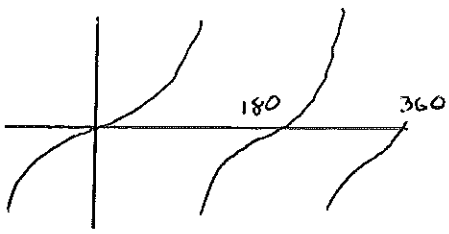
Let $\tan\theta = t$

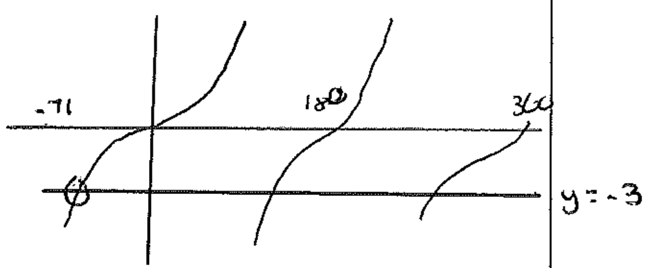
$\rightarrow t^2 + 3t = 0$

$t(t + 3) = 0$

$t = 0 \qquad t = -3$

or $\tan\theta = 0$ or $\tan\theta = -3$





$\theta = 180$

$\theta = \tan^{-1}(3) = -71.565...$

$\theta = 180 - 71.5 = 108.5$

$\theta = 360 - 71.5 = 288.5$

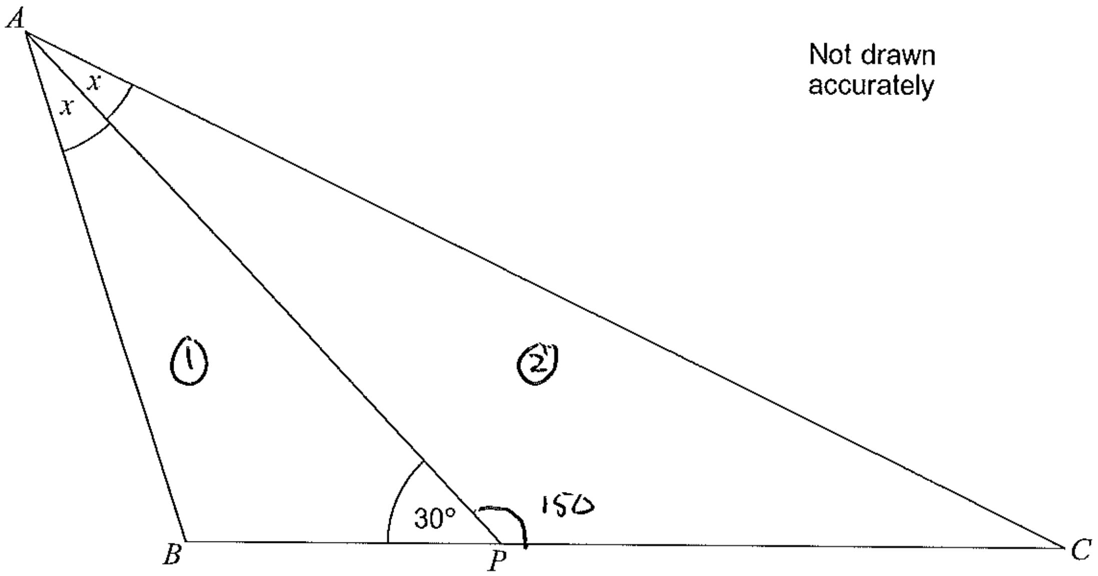
108.5, 180, 288.5

Answer *(5 marks)*

13

Turn over ▶

23 In triangle ABC, AP bisects angle BAC.



Use the sine rule in triangles ABP and ACP to prove that $\dfrac{AB}{AC} = \dfrac{BP}{PC}$

① $\dfrac{BP}{\sin(x)} = \dfrac{AB}{\sin(30)} \rightarrow \dfrac{BP}{\sin(x)} = \dfrac{AB}{1/2}$

② $\dfrac{PC}{\sin(x)} = \dfrac{AC}{\sin(150)} \rightarrow \dfrac{PC}{\sin(x)} = \dfrac{AC}{1/2}$

① $BP = \sin(x) \times AB/\tfrac{1}{2} \rightarrow BP = 2\sin(x)\,AB$

$\rightarrow BP/2AB = \sin(x)$

② $PC = \sin(x) \times AC/\tfrac{1}{2} \rightarrow PC = 2\sin(x)\,AC$

$\rightarrow PC/2AC = \sin(x)$

$\rightarrow BP/2AB = PC/2AC$

$\times 2AC$ $\quad \dfrac{2(AC)(BP)}{2AB} = PC$

$\times 2AB$ $\quad 2(AC)(BP) = 2(AB)(PC)$

$\div 2$ $\quad (AC)(BP) = (AB)(PC)$

Do not write outside the box

$\div AC$ $BP = \dfrac{(AB)(PC)}{AC}$

$\div PC$ $\dfrac{BP}{PC} = \dfrac{AB}{AC}$

(5 marks)

END OF QUESTIONS

5