## Write your name here



## Mathematics Iteration

## You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
- there may be more space than you need.
- You must show all your working.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- If your calculator does not have a $\pi$ button, take the value of $\pi$ to be 3.142 unless the question instructs otherwise.


## Information

- The total mark for this paper is 39 . There are 6 questions.
- Questions have been arranged in an ascending order of mean difficulty, as found by all students in the June 2017-November 2019 examinations.
- The marks for each question are shown in brackets - use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.


## 1MA1 Higher themed papers: Iteration

(a) Show that the equation $x^{3}+x=7$ has a solution between 1 and 2 .
(b) Show that the equation $x^{3}+x=7$ can be rearranged to give $x=\sqrt[3]{7-x}$
(c) Starting with $x_{0}=2$,
use the iteration formula $x_{n+1}=\sqrt[3]{7-x_{n}}$ three times to find an estimate for a solution of $x^{3}+x=7$

## 1MA1 Higher themed papers: Iteration

2 Using $\quad x_{n+1}=-2-\frac{4}{x_{n}^{2}}$
with $x_{0}=-2.5$
(a) find the values of $x_{1}, x_{2}$ and $x_{3}$

$$
\begin{aligned}
& x_{1}=\ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~
\end{aligned}
$$

(b) Explain the relationship between the values of $x_{1}, x_{2}$ and $x_{3}$ and the equation $x^{3}+2 x^{2}+4=0$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## 1MA1 Higher themed papers: Iteration

(a) Show that the equation $x^{3}+7 x-5=0$ has a solution between $x=0$ and $x=1$
(b) Show that the equation $x^{3}+7 x-5=0 \quad$ can be arranged to give $x=\frac{5}{x^{2}+7}$
(c) Starting with $x_{0}=1$, use the iteration formula $x_{n+1}=\frac{5}{x_{n}^{2}+7}$ three times to find an estimate for the solution of $x^{3}+7 x-5=0$

## 1MA1 Higher themed papers: Iteration

(d) By substituting your answer to part (c) into $x^{3}+7 x-5$, comment on the accuracy of your estimate for the solution to $x^{3}+7 x-5=0$
$\qquad$
$\qquad$

## 1MA1 Higher themed papers: Iteration

(a) Show that the equation $x^{3}+5 x-4=0$ has a solution between $x=0$ and $x=1$
(b) Show that the equation $x^{3}+5 x-4=0$ can be arranged to give $x=\frac{4}{x^{2}+5}$
(c) Starting with $\quad x_{0}=0$, use the iteration formula $\quad x_{n+1}=\frac{4}{x_{n}^{2}+5}$ twice, to find an estimate for the solution of $x^{3}+5 x-4=0$

## 1MA1 Higher themed papers: Iteration

5 (a) Show that the equation $x^{3}-3 x^{2}+3=0$ has a solution between $x=2$ and $x=3$
(2)
(b) Show that the equation $x^{3}-3 x^{2}+3=0$ can be rearranged to give $x=\sqrt[3]{3 x^{2} \quad 3}$
(c) Starting with $x_{0}=2$, use the iteration formula $x_{n+1}=\sqrt[3]{3 x^{2}} \quad 3$ to find the value of $x_{2}$. Give your answer correct to 3 decimal places.

## 1MA1 Higher themed papers: Iteration

6 (a) Show that the equation $3 x^{2}-x^{3}+3=0$ can be rearranged to give

$$
x=3+\frac{3}{x^{2}}
$$

(b) Using

$$
x_{\mathrm{n}+1}=3+\frac{3}{x_{n}^{2}} \quad \text { with } x_{0}=3.2
$$

find the values of $x_{1}, x_{2}$ and $x_{3}$
(c) Explain what the values of $x_{1}, x_{2}$ and $x_{3}$ represent.
$\qquad$
$\qquad$

