Name:

Level 2 Further Maths

Gradient of a Curve

Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. A curve has gradient function $\dfrac{dy}{dx} = 4x^2 + 1$

   Work out the gradient of the curve when $x = 3$

   $$4 \times 3^2 + 1$$
   $$4 \times 9 + 1$$
   $$36 + 1$$

   _____37_____

   (2)

2. A curve has gradient function $\dfrac{dy}{dx} = 15 - x^3$

   Work out the gradient of the curve when $x = -3$

   $$15 - (-3)^3$$
   $$15 - (-27)$$

   _____42_____

   (2)

3. A curve has gradient function $\dfrac{dy}{dx} = 7x^2 - 3$

   Work out the values of x for which the rate of change of y with respect to x is 25

   $$7x^2 - 3 = 25$$
   $$7x^2 = 28$$
   $$x^2 = 4$$

   _____$x = 2$ or $x = -2$_____

   (2)

4. A curve has gradient function $\dfrac{dy}{dx} = 5x - x^2$

(a) Work out the gradient of the curve when $x = 9$

$$5(9) - 9^2$$
$$45 - 81$$

.... $- 36$ ....

(2)

(b) Work out the values of x for which the rate of change of y with respect to x is 1

$$5x - x^2 = 1$$

$$\left(x - \tfrac{5}{2}\right)^2 = \tfrac{21}{4}$$

$$0 = x^2 - 5x + 1$$

$$x - \tfrac{5}{2} = \pm \tfrac{\sqrt{21}}{2}$$

$$\left(x - \tfrac{5}{2}\right)^2 - \tfrac{25}{4} + 1 = 0$$

$$x = \tfrac{5}{2} \pm \tfrac{\sqrt{21}}{2}$$

$$\left(x - \tfrac{5}{2}\right)^2 - \tfrac{21}{4} = 0$$

$$x = \tfrac{5}{2} + \tfrac{\sqrt{21}}{2} \text{ or } x = \tfrac{5}{2} - \tfrac{\sqrt{21}}{2}$$

(2)

5. $y = 2x^3 + 4x^2 - 7x$

(a) Find $\dfrac{dy}{dx}$

.... $6x^2 + 8x - 7$ ....

(2)

(b) Work out the gradient of $y = 2x^3 + 4x^2 - 7x$ at the point $(1, -1)$

$$x = 1$$

$$6(1)^2 + 8(1) - 7$$
$$6 + 8 - 7 = 7$$

.... 7 ....

(2)

6. Work out the gradient of the curve $y = 3x^2 - 4x + 7$ at the point $(-2, 27)$

$$\frac{dy}{dx} = 6x - 4$$

$$6 \times (-2) - 4$$

$$-12 - 4 = -16$$

..........-16..........

(3)

7. Work out the gradient of the curve $y = (x - 2)(3x + 1)$ at the point when $x = 3$

$$y = 3x^2 - 5x - 2$$

$$\frac{dy}{dx} = 6x - 5$$

$$6(3) - 5 = 13$$

..........13..........

(3)

8. Work out the gradient of the curve $y = x^3(8 - x)$ at the point on the curve where $x = -1$

$$y = 8x^3 - x^4$$

$$\frac{dy}{dx} = 24x^2 - 4x^3$$

when $x = -1$ $\frac{dy}{dx} = 24(-1)^2 - 4(-1)^3$

$= 24(1) + 4$

$= 28$

..........28..........

(3)

9. $y = \dfrac{3}{5}x^5 - 3x^3$

Work out the rate of change of y with respect to x when $x = -1$

$$\frac{dy}{dx} = 3x^4 - 9x^2$$

when $x = -1$

$$\frac{dy}{dx} = 3 - 9$$

-6

(3)

10. $y = \dfrac{2x^6 - x^5}{x^3}$

Work out the rate of change of y with respect to x when $x = 3$

$$y = 2x^3 - x^2$$

$$\frac{dy}{dx} = 6x^2 - 2x$$

when $x = 3$

$$\frac{dy}{dx} = 6(3)^2 - 2(3)$$

$$= 54 - 6$$

48

(3)

11. Work out the gradient of the curve $y = (x - 2)(x + 1)^2$ at the point $(2, 0)$

$$y = (x - 2)(x^2 + 2x + 1)$$
$$y = x^3 + 2x^2 + x - 2x^2 - 4x - 2$$
$$y = x^3 - 3x - 2$$

$$\frac{dy}{dx} = 3x^2 - 3$$

when $x = 2$

$$\frac{dy}{dx} = 12 - 3$$

................9.................

(4)

12. A curve has equation $y = 2x^2 - 3x + 1$

The gradient of the curve at point P is 9

Work out the coordinates of the point P.

$$\frac{dy}{dx} = 4x - 3$$

$$4x - 3 = 9$$
$$4x = 12$$
$$x = 3$$

$$(3, 10)$$

when $x = 3$
$$y = 2 \times 3^2 - 3 \times 3 + 1$$
$$y = 2 \times 9 - 9 + 1$$
$$y = 18 - 9 + 1$$
$$y = 10$$

.............$(3, 10)$.............

(4)

13. A curve has equation $y = (x + 2)(x - 3)$

The gradient of the curve at point P is –4

Work out the coordinates of the point P.

$$y = x^2 - x - 6$$

$$\frac{dy}{dx} = 2x - 1$$

$$2x - 1 = -4$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

$$y = (-1.5 + 2)(-1.5 - 3)$$

$$y = 0.5 \times -4.5$$

$$= -2.25$$

$$\left(-\frac{3}{2}, -\frac{9}{4}\right)$$

(4)

14. A curve has equation $y = \frac{2}{3}x^3$

The gradient of the curve at the points P and Q are equal to 18

Work out the coordinates of the points P and Q.

$$\frac{dy}{dx} = 2x^2$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

$$y = \frac{2}{3} \times 3^3 \qquad \text{or} \qquad y = \frac{2}{3} \times (-3)^3$$

$$y = \frac{2}{3} \times 27 \qquad\qquad y = \frac{2}{3} \times (-27)$$

$$y = 18 \qquad\qquad\qquad y = -18$$

$$(3, 18) \quad (-3, -18)$$

(6)

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15. A curve has the equation $y = x^2 + ax + 4$ where a is a constant.

The gradient of the curve when x = 2 is twice the gradient of the curve when x = −1

Work out the value of a

$$\frac{dy}{dx} = 2x + a$$

when $x = 2$

$$\frac{dy}{dx} = 4 + a$$

when $x = -1$

$$\frac{dy}{dx} = -2 + a$$

$$4 + a = 2(-2 + a)$$
$$4 + a = -4 + 2a$$
$$8 = a$$

$a = 8$

....

(4)

16. A curve has the equation $y = x^3 + ax^2 - 8$ where a is a constant.

The gradient of the curve when x = 2 is eleven times the gradient of the curve when x = −2

Work out the value of a

$$\frac{dy}{dx} = 3x^2 + 2ax$$

when $x = 2$

$$\frac{dy}{dx} = 12 + 4a$$

when $x = -2$

$$\frac{dy}{dx} = 12 - 4a$$

$$11(12 - 4a) = 12 + 4a$$
$$132 - 44a = 12 + 4a$$
$$120 = 48a$$
$$a = 2.5$$

$a = 2.5$

....

(5)