| Centre Number | | | | | | Candidate Number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Surname | MR BARTON'S | | | | | | | | | |
| Other Names | SOLUTIONS | | | | | | | | | |
| Candidate Signature | | | | | | | | | | |

| For Examiner's Use | |
|---|---|
| Examiner's Initials | |
| Pages | Mark |
| 3 | |
| 4 – 5 | |
| 6 – 7 | |
| 8 – 9 | |
| 10 – 11 | |
| 12 – 13 | |
| 14 – 15 | |
| 16 – 17 | |
| 18 – 19 | |
| 20 – 21 | |
| 22 – 23 | |
| 24 – 25 | |
| 26 – 27 | |
| 28 | |
| TOTAL | |

AQA

Level 2 Certificate in Further Mathematics
June 2015

# Further Mathematics 8360/2

## Level 2

**Paper 2 Calculator**

**Friday 19 June 2015 9.00 am to 11.00 am**

**For this paper you must have:**
- a calculator
- mathematical instruments.

**Time allowed**
- 2 hours

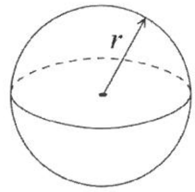
**Instructions**
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

**Formulae Sheet**

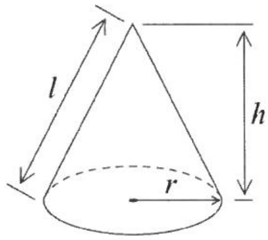
**Volume of sphere** $= \frac{4}{3}\pi r^3$

**Surface area of sphere** $= 4\pi r^2$

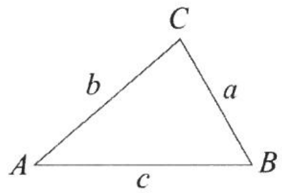



**Volume of cone** $= \frac{1}{3}\pi r^2 h$

**Curved surface area of cone** $= \pi r l$




**In any triangle** $ABC$




**Area of triangle** $= \frac{1}{2}ab\sin C$

**Sine rule** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

**Cosine rule** $a^2 = b^2 + c^2 - 2bc\cos A$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

**The Quadratic Equation**

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

**Trigonometric Identities**

$\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$ $\sin^2\theta + \cos^2\theta \equiv 1$

Answer **all** questions in the spaces provided.

**1** A circle, centre (0, 0), has circumference $12\pi$

Work out the equation of the circle.

**[2 marks]**

$C = \pi d$

$\rightarrow 12\pi = \pi d$

$\rightarrow 12 = d$ centre $= (0,0)$

$\rightarrow$ radius $= 6$

Answer $x^2 + y^2 = 6^2$

**2** $a : b : c = 5 : 3 : 2$

Work out $4a - c : 3b$

Give your answer in its simplest form.

**[2 marks]**

Let 1 part $= x$

$\rightarrow 4(5) - 2 : 3(3)$

$18 : 9$

$2 : 1$

Answer 2 : 1

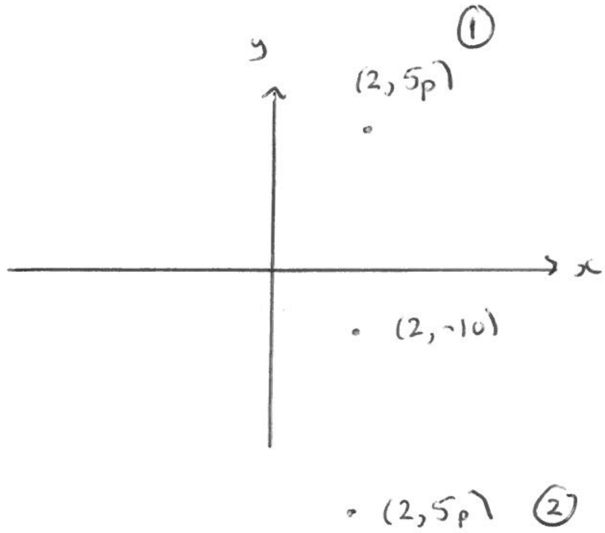
4

**Turn over ▶**

**3** The distance between the points $(2, 5p)$ and $(2, -10)$ is 30 units.

Work out the **two** possible values of $p$.

**[3 marks]**




1 $5p - (-10) = 30$ 2 $-10 - 5p = 30$

$5p + 10 = 30$ $-10 = 30 + 5p$

$5p = 20$ $-40 = 5p$

$p = 4$ $p = -8$

Answer 4 and -8

Do not write outside the box

**4** The first term of a sequence is $1 - a$

The term-to-term rule of a sequence is

add $2a$ then multiply by 3

**4 (a)** Show that the second term is $3 + 3a$

**[1 mark]**

$[(1 - a) + 2a] \times 3$

$[1 + a] \times 3 \rightarrow 3 + 3a$

**4 (b)** The third term is 16

Work out the value of $a$.

**[3 marks]**

$[(3 + 3a) + 2a] \times 3 = 16$

$3[3 + 5a] = 16$

$9 + 15a = 16$

$15a = 7$

$a = 7/15$

Answer $7/15$

7

**Turn over ▶**

Do not write outside the box

**5** A straight line L

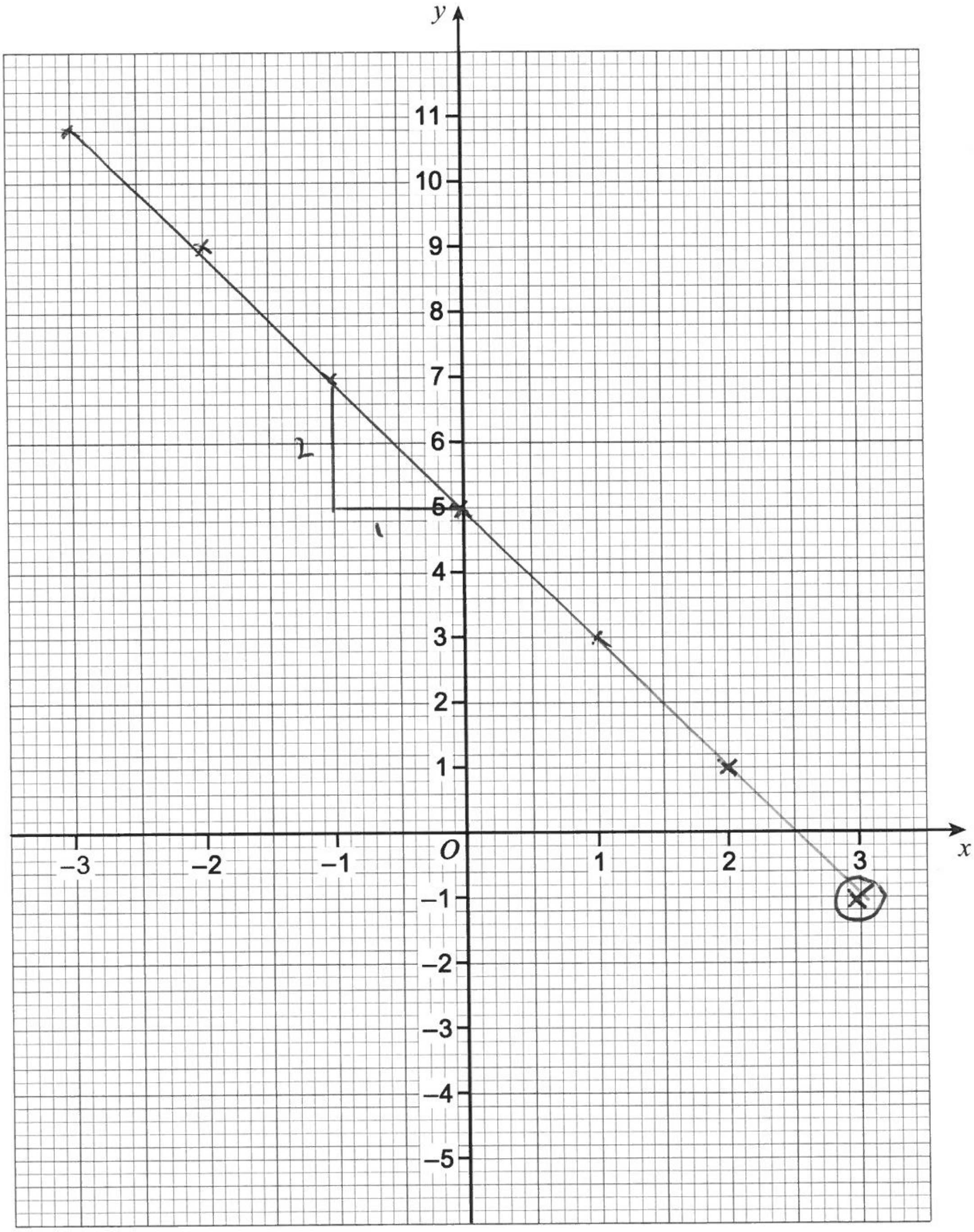
is parallel to the straight line $y = 1 - 2x$
passes through $(3, -1)$

On the grid below, draw the straight line L for values of $x$ from $-3$ to 3.

**[4 marks]**

Gradient = −2

→ Each 1 across = 2 down



Do not write outside the box

**6** Write $\dfrac{15x^8 - 18x^7}{3x^2}$ in the form $ax^n - nx^a$ where $a$ and $n$ are integers.

**[2 marks]**

$= \dfrac{5x^8 - 6x^7}{x^2} \quad = 5x^6 - 6x^5$

**7** $y = \frac{2}{3}x^6 - 8x^3$

Work out the rate of change of $y$ with respect to $x$ when $x = -1$

**[3 marks]**

$dy/dx = \frac{12}{3}x^5 - 24x^2$

$= 4x^5 - 24x^2$

when $x = -1 \Rightarrow dy/dx = 4(-1)^5 - 24(-1)^2$

$= -4 - 24$

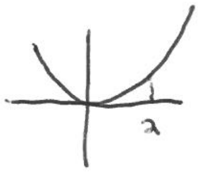
$= -28$

Answer $-28$

9

**8 (a)** $f(x) = x^4$
The domain of $f(x)$ is $x \geqslant 2$

Work out the range of $f(x)$.



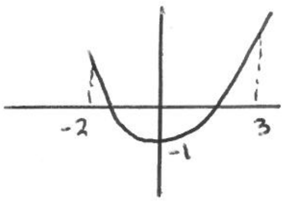

**[1 mark]**

$2^4 = 16$

Answer $f(x) \geqslant 16$

**8 (b)** $g(x) = x^2 - 1$
The domain of $g(x)$ is $-2 \leqslant x \leqslant 3$

Work out the range of $g(x)$.




**[2 marks]**

Highest when $x = 3$

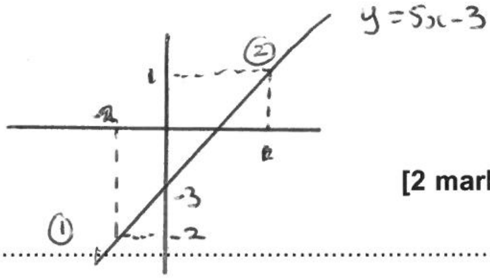
$\rightarrow f(3) = 3^2 - 1 = 8$

Lowest at minimum point $= -1$

Answer $-1 \leq f(x) \leq 8$

**8 (c)** $h(x) = 5x - 3$
The **range** of $h(x)$ is $-2 < h(x) < 1$

Work out the domain of $h(x)$.




**[2 marks]**

(1) $5x - 3 = -2$ (2) $5x - 3 = 1$

$5x = 1$ $5x = 4$

$x = 1/5$ $x = 4/5$

Answer $\frac{1}{5} < x < \frac{4}{5}$

Do not write outside the box

**9 (a)** Solve $6(2y-3)-10=2y$

**[3 marks]**

$12y - 18 - 10 = 2y$

$12y - 28 = 2y$

$10y - 28 = 0$

$10y = 28$

$y = 28/10 = 2.8$

$y =$ 2.8

**9 (b)** Solve $\dfrac{\sqrt{w+4}}{2} = 6$

**[3 marks]**

$\times 2$ $\sqrt{w+4} = 12$

$^2$ $w + 4 = 144$

$-4$ $w = 140$

$w =$ 140

**9 (c)** Solve $3m^{\frac{1}{5}} + 9 = 0$

**[2 marks]**

$-9$ $3m^{\frac{1}{5}} = -9$

$\div 3$ $m^{\frac{1}{5}} = -3$

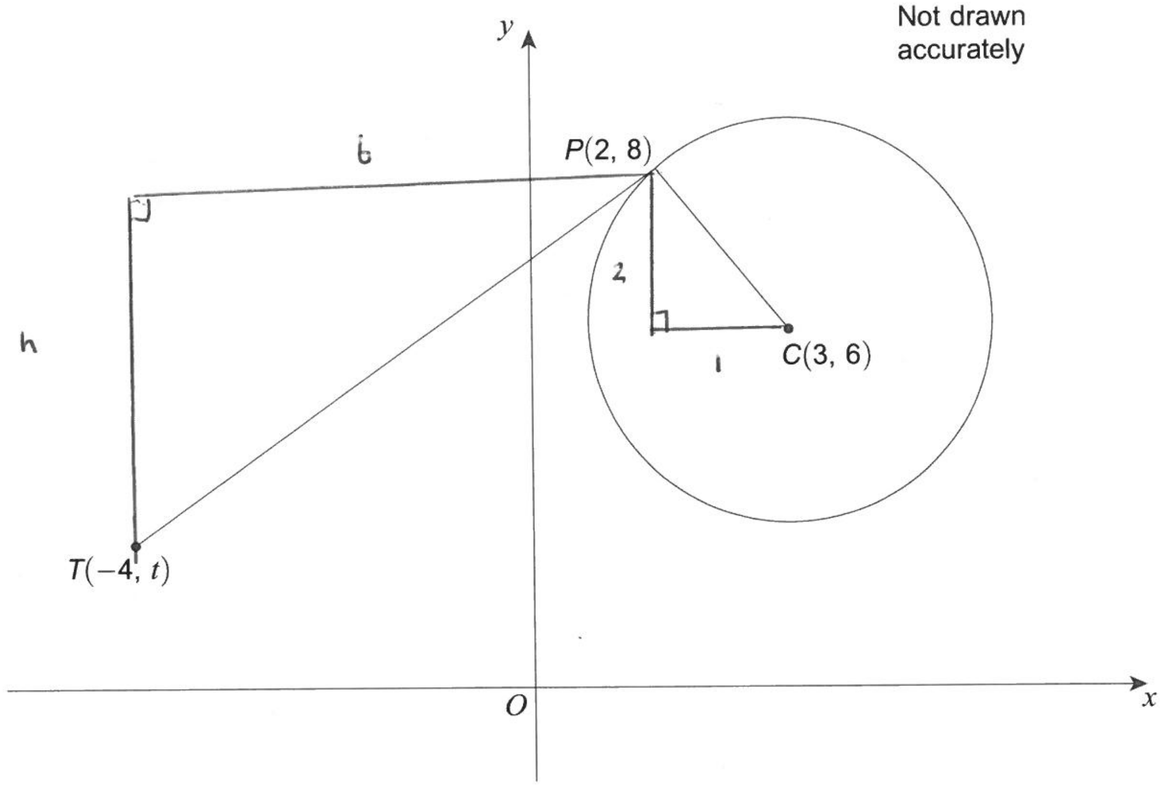
power 5 $m = (-3)^5 = -243$

$m =$ -243

13

**Turn over ▶**

**10** The diagram shows a circle, centre $C$.
$TP$ is a tangent to the circle at $P$.




Work out the value of $t$.

**[4 marks]**

Gradient $CP = \frac{-2}{1} = -2$

Tangent is perpendicular to radius

$\rightarrow$ Gradient of $PT = \frac{1}{2}$

$\frac{h}{6} = \frac{1}{2} \rightarrow h = 3$

$\boxed{y}$ $8 - 3 = 5$

Answer $t = 5$

Do not write outside the box

**11 (a)** Expand and simplify $(3w + 2y)(w - 4y)$

**[3 marks]**

$3w^2 - 12wy + 2wy - 8y^2$

$3w^2 - 10wy - 8y^2$

Answer ..........................................................................

**11 (b)** Expand and simplify $\frac{3}{x^2}\left(\frac{x}{3} + 3x^2 - 1\right)$

**[3 marks]**

$\frac{3x}{3x^2} + \frac{9x^2}{x^2} - \frac{3}{x^2}$

$= \frac{x}{x^2} + \frac{9x^2}{x^2} - \frac{3}{x^2}$

Answer ..........................................................................

$= \frac{x + 9x^2 - 3}{x^2}$

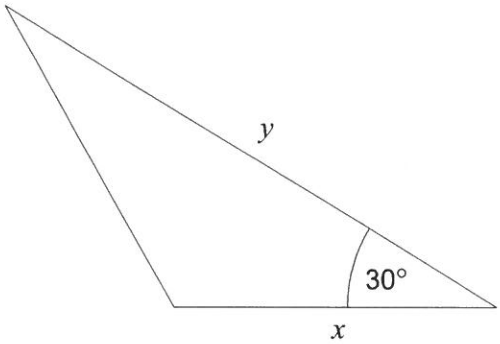
10

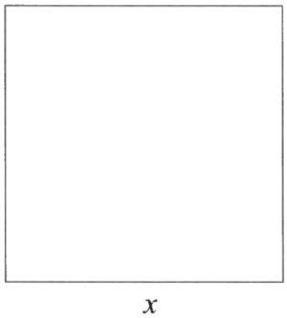
**Turn over ▶**

Do not write outside the box

**12** The area of the triangle is equal to the area of the square.
All dimensions are in centimetres.

Not drawn accurately




Write $y$ in terms of $x$.

**[2 marks]**

Square Area = $x^2$

Triangle Area = $\frac{1}{2}ab\sin(c)$

$= \frac{1}{2}xy\,\frac{1}{2} = \frac{1}{4}xy$

$\rightarrow \frac{1}{4}xy = x^2$

$\cancel{4x}\; xy = 4x^2$

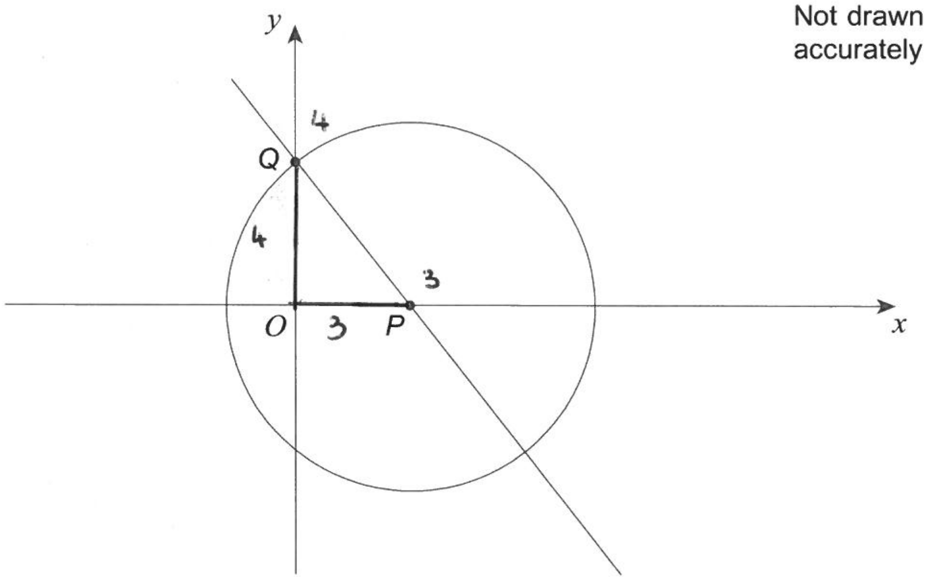
$y = 4x$

Answer ..............................................................

Do not write outside the box

**13** The diagram shows a circle, centre $P$, and a straight line passing through points $P$ and $Q$.
$Q$ lies on the $y$-axis and on the circumference of the circle.

The equation of the circle is $(x-3)^2 + y^2 = 25$




Not drawn accurately

Work out the equation of the straight line through $P$ and $Q$.
Give your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

**[4 marks]**

$\boxed{P}$ = centre = (3, 0)

$\boxed{Q}$ → $x = 0$ → $(0-3)^2 + y^2 = 25$

$9 + y^2 = 25$

→ $y^2 = 16$

→ $y = 4$ or $-4$

→ $Q = (0, 4)$

Gradient = $-\frac{4}{3}$

$x_1 = 3$

$y_1 = 0$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{4}{3}(x-3)$

$3y = -(x-3)$

$3y = -4x + 12$

$4x + 3y - 12 = 0$

Answer ..........................................................................

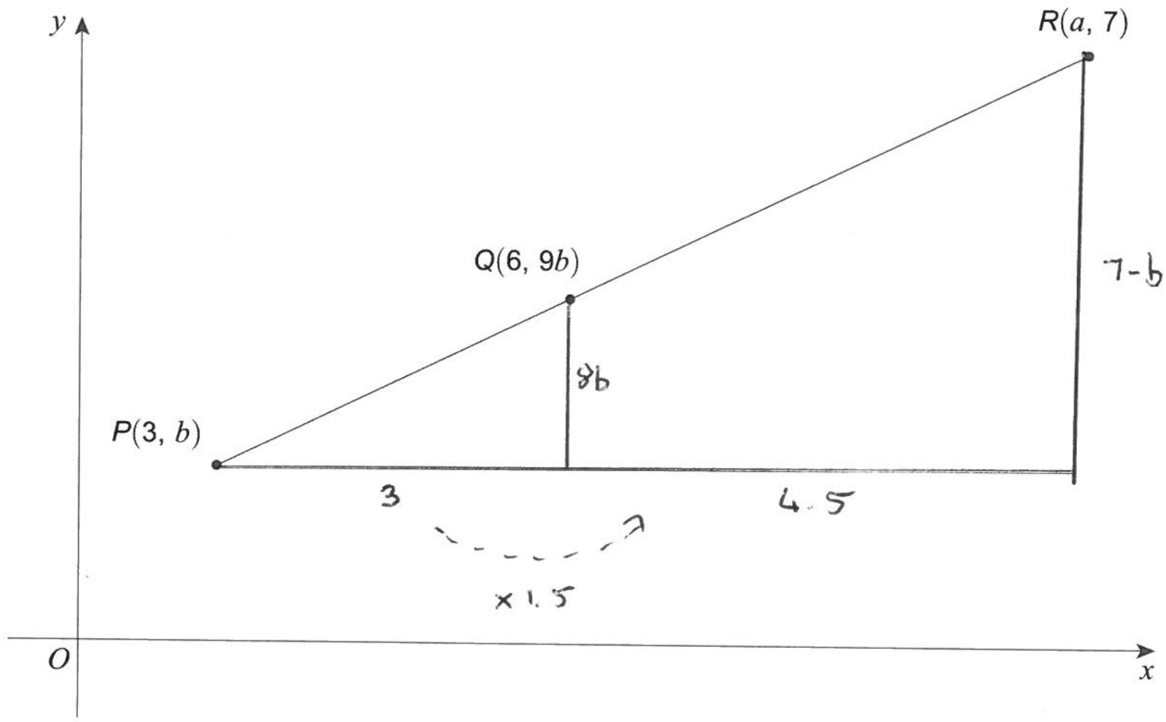
6

Turn over ▶

Do not write outside the box

**14** $PQR$ is a straight line.
$PQ : QR$ is $2 : 3$

Not drawn accurately




**14 (a)** Show that $a = 10.5$

**[2 marks]**

PQ : QR
= 2 : 3
= 1 : 1.5
→ scale factor = 1.5

x value for QR
= 3 × 1.5 = 4.5
→ Horizontal distance
= 3 + 4.5 = 7.5
→ a = 3 + 7.5
= 10.5

**14 (b)** Work out the value of $b$.

**[3 marks]**

Gradient of PQ = PR

$\rightarrow \frac{8b}{3} = \frac{7-b}{7.5}$

$7.5(8b) = 3(7-b)$

$60b = 21 - 3b$

$63b = 21$

$b = 21/63 = 1/3$

Answer $b = \frac{1}{3}$

**15** Use algebra to prove that the value of $\frac{8c^2+16}{3c^2+6}+\frac{1}{3}$ is an integer for all values of $c$.

**[3 marks]**

~~Need common denominator:~~

FACTORISE

Simplify 1st Fraction: $\frac{8c^2+16}{3c^2+6} = \frac{8(c^2+2)}{3(c^2+2)}$

$= 8/3$

Now add: $\frac{8}{3}+\frac{1}{3} = \frac{9}{3} = 3$

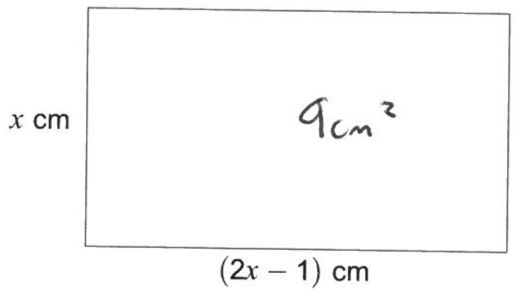
Always an integer!

8

**Turn over ▶**

Do not write outside the box

**16** The diagram shows a rectangle with area 9 $cm^2$




Not drawn accurately

Set up and solve an equation to work out the value of $x$.
Give your answer to 3 significant figures.

**[5 marks]**

$$x(2x-1) = 9$$

$$2x^2 - x = 9$$

$$2x^2 - x - 9 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 2 \times (-9)}}{2 \times 2}$$

$a = 2$
$b = -1$
$c = -9$

$$x = \frac{1 \pm \sqrt{73}}{4} \Rightarrow x = 2.3860\ldots$$

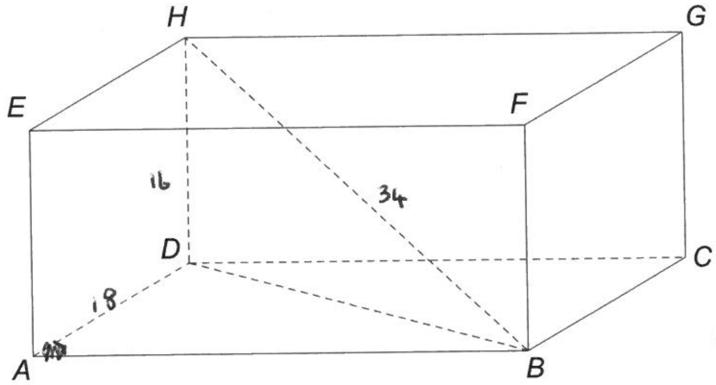
or $x = -1.8860\ldots$

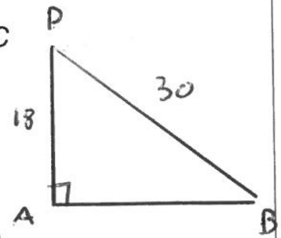
Must be positive!

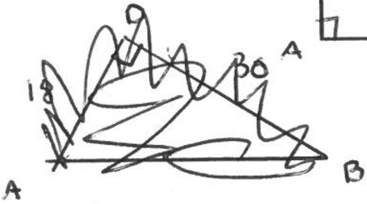
$x =$ 2.39

Do not write outside the box

**17** *ABCDEFGH* is a cuboid.



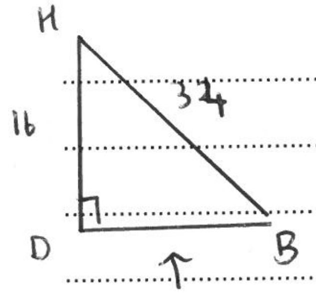



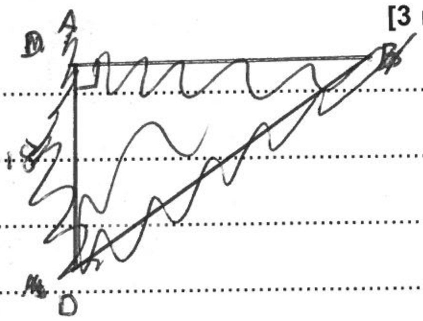




$HB = 34$ cm
$HD = 16$ cm
$AD = 18$ cm

**17 (a)** Work out the length of *AB*.

**[3 marks]**




$DB = \sqrt{34^2 - 16^2}$

$= \sqrt{900}$

$= 30$

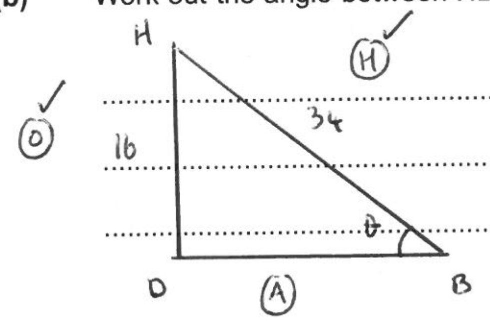
$AB = \sqrt{30^2 - 18^2}$

$= \sqrt{576}$

Answer 24 cm

**17 (b)** Work out the angle between *HB* and *ABCD*.

**[2 marks]**




$\sin(\theta) = \frac{O}{H}$

$\theta = \sin^{-1}\left(\frac{16}{34}\right)$

Answer 28.072... degrees

10

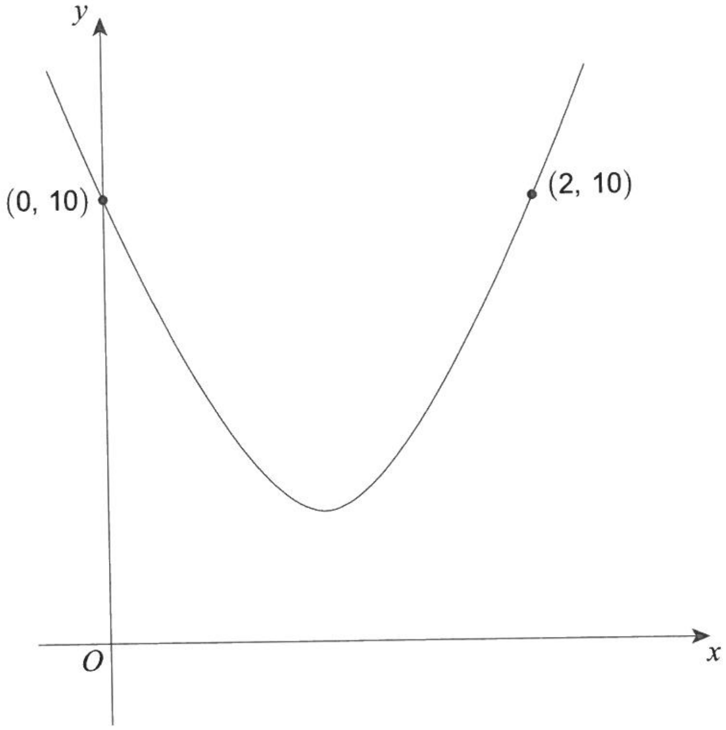
**Turn over ▶**

Do not write outside the box

**18** The sketch shows the quadratic curve $y = 4(x - a)^2 + b$

The curve passes through (0, 10) and (2, 10)

Not drawn accurately




**18 (a)** Give reasons why the value of $a$ is 1.

**[2 marks]**

The curve ~~ha~~ has a vertical line of symmetry

through the minimum point

$\rightarrow \frac{0+2}{2} \Rightarrow x = 1 \rightarrow a = 1$

Do not write outside the box

**18 (b)** Work out the value of $b$. $y = 4(x-1)^2 + b$ **[2 marks]**

when $x = 0$, $y = 10$

$\rightarrow 10 = 4(0-1)^2 + b$

$10 = 4(-1)^2 + b$

$10 = 4 + b \rightarrow b = 6$

Answer $b = 6$

**18 (c)** Write the equation of the curve in the form $y = px^2 + qx + r$ **[2 marks]**

$y = 4(x-1)^2 + 6$

$y = 4[(x-1)(x-1)] + 6$

$\rightarrow y = 4(x^2 - 2x + 1) + 6$

$\rightarrow y = 4x^2 - 8x + 4 + 6$

Answer $y = 4x^2 - 8x + 10$

**19** Use the factor theorem to show that $(x - 3)$ is **not** a factor of $x^3 - 10x - 3$ **[2 marks]**

$f(x) = x^3 - 10x - 3$

$f(3) = (3)^3 - 10(3) - 3$

$= 27 - 30 - 3 = -6$

$-6 \neq 0$

$\therefore (x-3)$ is NOT a factor

8

**Turn over ▶**

Do not write outside the box

**20 (a)** The transformation matrix **P** represents a 90° anti-clockwise rotation about the origin.

Describe fully the **single** transformation represented by the matrix $\mathbf{P}^3$

**[2 marks]**

$P^3$ = P repeated 3 times

→ Rotation 270° anti-clockwise about the origin.

**20 (b)** The transformation matrix **Q** is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

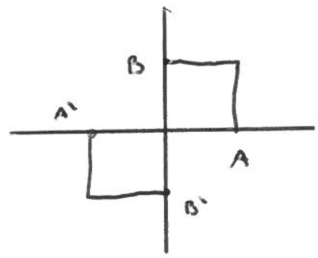
The transformation matrix **R** is $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Describe fully the **single** transformation represented by the matrix **QR**.

**[2 marks]**

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$




Rotation 180° about (0,0)

Do not write outside the box

**21** A cubic curve has

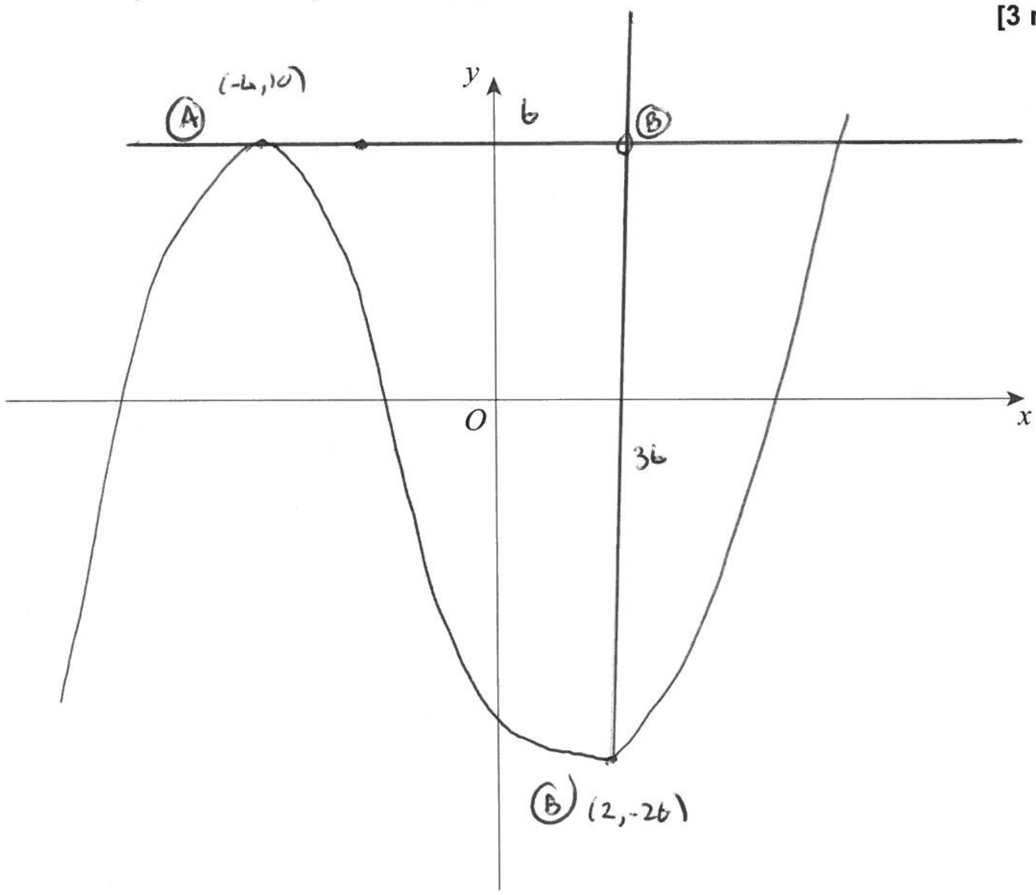
a maximum point at $A\ (-4,\ 10)$

a minimum point at $B\ (2,\ -26)$

The tangent to the curve at $A$ and the normal to the curve at $B$ intersect at point $C$.

Work out the area of triangle $ABC$.
You may sketch a diagram to help you.

**[3 marks]**




B must be (2, 10)

Base = 2 - - 4 = 6

Height = 10 - - 26 = 36

Area = $\dfrac{6 \times 36}{2}$ = 108

Answer 108 square units

7

**Turn over ▶**

Do not write outside the box

**22** A quadratic sequence starts

302 600 894 1184 ..........

**22 (a)** Work out an expression for the $n$th term.

**[3 marks]**

1st diff 298 294 290

2nd diff $-4$ $-4$ $\rightarrow$ $-2x^2$

302 600 894 1184

$\boxed{-2n^2}$ $-2$ $-8$ $-18$ $-32$

304 608 912 1216

304 304 304

$\boxed{304n}$ 304 608 912 1216

Answer $-2n^2 + 304n$

**22 (b)** A term in the sequence has value 0

Find the position of this term.

**[2 marks]**

$-2n^2 + 304n = 0$

$\div -2$ $n^2 - 152n = 0$

$n(n - 152) = 0$

$n = 0$ $n = 152$

Answer 152nd position

Do not write outside the box

**23** The continuous curve $y = \mathrm{f}(x)$ has exactly **two** stationary points.

*P* is a maximum point when $x = a$

*Q* is a stationary point of inflection when $x = b$

$a < b$

Which of these is correct?

Tick **one** box only.

**[1 mark]**

| | |
|---|---|
| When $a < x < b$, $\frac{dy}{dx}$ is positive **and** when $x > b$, $\frac{dy}{dx}$ is positive | ☐ |
| When $a < x < b$, $\frac{dy}{dx}$ is positive **and** when $x > b$, $\frac{dy}{dx}$ is negative | ☐ |
| When $a < x < b$, $\frac{dy}{dx}$ is negative **and** when $x > b$, $\frac{dy}{dx}$ is positive | ☐ |
| When $a < x < b$, $\frac{dy}{dx}$ is negative **and** when $x > b$, $\frac{dy}{dx}$ is negative | ✓ |

6

Do not write outside the box

**24** $a^2 < 4$ and $a + 2b = 8$

Work out the range of possible values of $b$.
Give your answer as an inequality.

**[4 marks]**

$a^2 < 4 \rightarrow -2 < a < 2$

If $a = -2$ $\quad a + 2b = 8$

$\rightarrow -2 + 2b = 8$

$\rightarrow 2b = 10 \rightarrow b = 5$

If $a = 2$ $\quad a + 2b = 8$

$\rightarrow 2 + 2b = 8$

$2b = 6$

$b = 3$

Answer $3 < b < 5$

Do not write outside the box

**25** Work out the values of $x$ between 0° and 360° for which

$$25\cos^2 x = 9$$

Give your answers to 1 decimal place.

**[4 marks]**

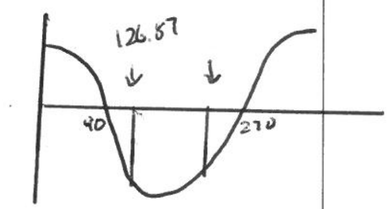
$\cos^2(x) = \frac{9}{25}$

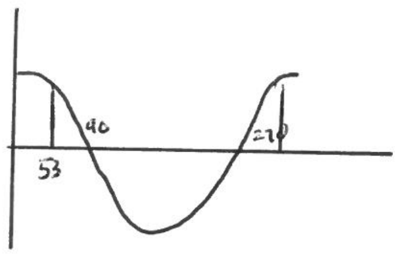
$\cos(x) = \pm\sqrt{9/25}$

$\Rightarrow \cos(x) = 3/5$ or $\cos(x) = -3/5$

$x = \cos^{-1}(3/5)$ $\quad x = \cos^{-1}(-3/5)$

$= 53.13°$ $\quad = 126.87°$




$x = 53.13°$ or $360 - \cancel{270} 53.13°$

$= 306.87°$

$x = 126.87$

or

$x = 360 - 126.87$

$= 233.13°$

Answer $53.1°, 306.9°, 126.9°, 233.1°$

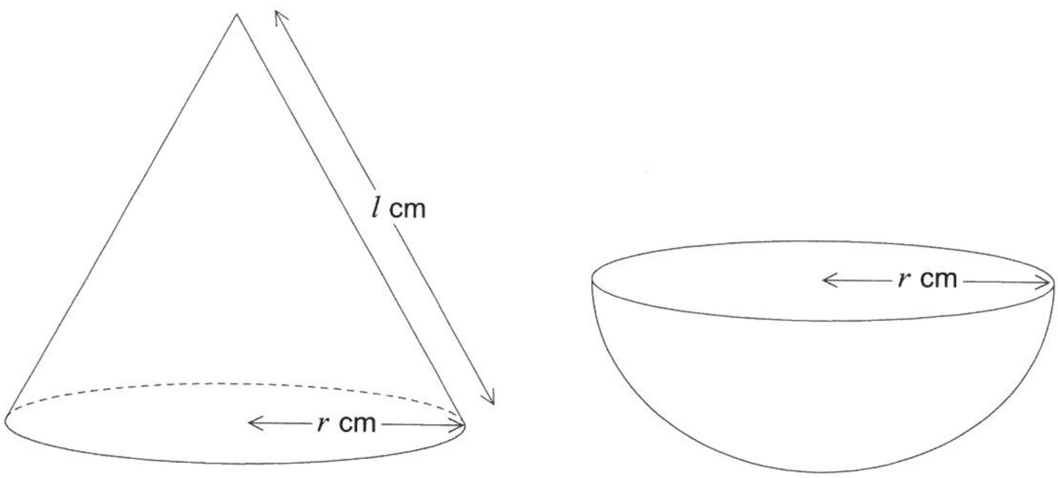
8

**Turn over ▶**

Do not write outside the box

**26** A cone has base radius $r$ cm and slant height $l$ cm

A hemisphere has radius $r$ cm




**26 (a)** The curved surface area of the cone equals the curved surface area of the hemisphere.

Show that $l = 2r$

**[1 mark]**

SA of cone = $\pi r l$

SA of hemisphere = $\frac{1}{2}[4\pi r^2] = 2\pi r^2$

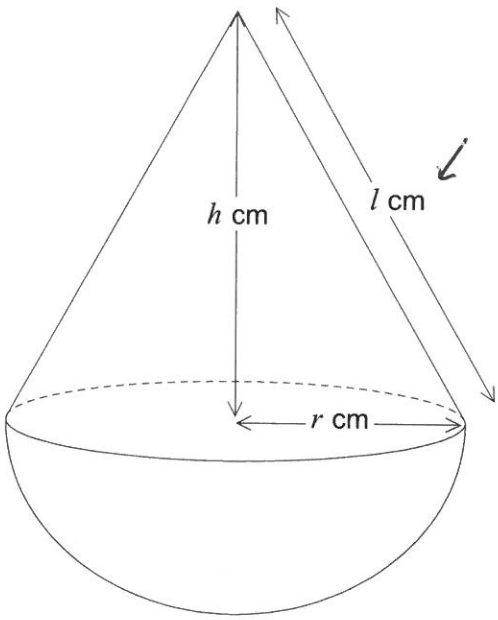
$$\pi r l = 2\pi r^2$$

$\div\pi$ $\quad r l = 2r^2$

$\div r$ $\quad l = 2r$

**26 (b)** The cone has vertical height $h$ cm

The cone and hemisphere are joined to make the shape shown below.




$l = 2r$

Show that the volume of the shape can be written as

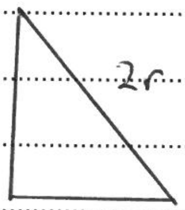
$\frac{1}{3}\pi r^3(a + \sqrt{b})$ cm$^3$ where $a$ and $b$ are integers.

**[4 marks]**

$\boxed{\text{HEMISPHERE}}$ $V = \frac{1}{2}\left[\frac{4}{3}\pi r^3\right]$

$= \frac{2}{3}\pi r^3$

$\boxed{\text{CONE}}$ Need height:




$h = \sqrt{(2r)^2 - r^2}$

$= \sqrt{3r^2}$

$= \sqrt{3}r$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi r^2 (\sqrt{3}r)$

$= \sqrt{3}/3 \, \pi r^3$

TOTAL VOL $= \frac{2}{3}\pi r^3 + \sqrt{3}/3 \, \pi r^3$

$= \frac{1}{3}\pi r^3 (2 + \sqrt{3})$

5

**Turn over ▶**

2 7

Do not write outside the box

**27** Work out the values of $a$ when

$$2^{a^2} = 8^a \times 16$$

Do **not** use trial and improvement.
You **must** show your working.

**[4 marks]**

$2^{a^2} = 8^a \times 16$

$2^{a^2} = (2^3)^a \times 2^4$

$2^{a^2} = 2^{3a} \times 2^4$

$2^{a^2} = 2^{3a+4}$

$\rightarrow a^2 = 3a + 4$

$-3a - 4$ $a^2 - 3a - 4 = 0$

$(a - 4)(a + 1) = 0$

$a = 4$ $a = -1$

Answer ..........................................................................

**END OF QUESTIONS**



4