

Name: _____

Level 2 Further Maths

Function Notation

Composite Functions

Inverse Functions



Corbettmaths

Ensure you have: Pencil or pen

Answers

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/



1. $f(x) = x^3 - 27$ for all values of x

$g(x) = x^2 - 25$ for all values of x

$h(x) = x^2 - 3x + 4$

(a) Work out the value of $f(5)$

$$f(5) = 5^3 - 27$$

98

.....
(1)

(b) Work out the value of $h(-2)$

$$h(-2) = (-2)^2 - 3(-2) + 4$$

14

.....
(1)

(c) Work out the value of x , such that $f(x) = 0$

$$x^3 - 27 = 0$$

$$x^3 = 27$$

$$x = 3$$

3

.....
(2)

(d) Work out the value of x , such that $g(x) = 11$

$$x^2 - 25 = 11$$

$$x^2 = 36$$

$$x = \pm 6$$

$x = 6$ or $x = -6$

.....
(2)

(e) Work out the value of x , such that $h(x) = 14$

$$x^2 - 3x + 4 = 14$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5 \text{ or } x = -2$$

.....
(3)

(f) Work out the value of x , such that $g(x) = h(x)$

$$x^2 - 25 = x^2 - 3x + 4$$

$$3x = 29$$

$$x = \frac{29}{3}$$

$$x = \frac{29}{3}$$

.....
(3)

2. $f(x) = \sin x$ for all values of x
 $g(x) = \cos x$ for all values of x

(a) Calculate the value of $f(630^\circ)$

$$\sin 630^\circ =$$

-1

.....
(1)

(b) Calculate the value of $g(-90^\circ)$

$$\cos(-90^\circ)$$

0

.....
(1)

3. $f(x) = 2x - 7$ for all values of x

Solve $f(x^2) = x - 1$

$$2x^2 - 7 = x - 1$$

$$2x^2 - x - 6 = 0$$

$$(2x + 3)(x - 2) = 0$$

$$x = -\frac{3}{2} \text{ or } x = 2$$

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(4)

4. $f(x) = (x + 7)(x - 3q)$ for all values of x

$g(x) = 2x^2 - 6x - 30$ for all values of x

(a) Given $f(0) = -42$

Show that $q = 2$

$$(0 + 7)(0 - 3q) = -42$$

$$-21q = -42$$

$$q = 2$$

(1)

(b) Solve $f(x) = g(x)$

$$(x + 7)(x - 6) = 2x^2 - 6x - 30$$

$$x^2 + x - 42 = 2x^2 - 6x - 30$$

$$0 = x^2 - 7x + 12$$

$$0 = (x - 3)(x - 4)$$

$$x = 3 \text{ or } x = 4$$

.....
(3)

5. $f(x) = 4x^4 - x^2$ for all values of x

Show that $f(5x) = ax^2(bx - 1)(bx + 1)$ where a and b are integers.

$$\begin{aligned} f(5x) &= 4(5x)^4 - (5x)^2 \\ &= 2500x^4 - 25x^2 \\ &= 25x^2(100x^2 - 1) \\ &= 25x^2(10x - 1)(10x + 1) \end{aligned}$$

.....
(3)

6. $f(x) = 8x - 7$

Find $f^{-1}(x)$

$$\begin{aligned} y &= 8x - 7 \\ y + 7 &= 8x \\ \frac{y + 7}{8} &= x \end{aligned}$$

$$f^{-1}(x) = \frac{x + 7}{8}$$

.....
(2)

7. $g(x) = 5 - \frac{2 - 3x}{4}$

Find $g^{-1}(x)$

$$\begin{aligned} y &= 5 - \frac{2 - 3x}{4} \\ \frac{2 - 3x}{4} &= 5 - y \end{aligned}$$

$$-3x = 18 - 4y$$

$$3x = 4y - 18$$

$$x = \frac{4y - 18}{3}$$

$$x = \frac{4}{3}y - 6$$

$$2 - 3x = 20 - 4y$$

$$g^{-1}(x) = \frac{4}{3}x - 6$$

.....
(2)

8. $f(x) = x + 7$

$$g(x) = 4x - 1$$

$$h(x) = x^2 - 2$$

(a) Find $gf(x)$

$$4(x + 7) - 1$$

$$4x + 28 - 1$$

$$\underline{gf(x) = 4x + 27}$$

(2)

(b) Find $hh(x)$

$$(x^2 - 2)^2 - 2$$

$$(x^2 - 2)(x^2 - 2) - 2$$

$$x^4 - 4x^2 + 4 - 2$$

$$\underline{hh(x) = x^4 - 4x^2 + 2}$$

(2)

(c) Work out the value of $fh(-3)$

$$h(-3) = (-3)^2 - 2 = 7$$

$$f(7) = 7 + 7$$

14

.....
(2)

9. $f(x) = x + 3$

$g(x) = x^2 - 25$ $(x + 3)^2 - 25 = 0$

Solve $gf(x) = 0$

$$x^2 + 6x + 9 - 25 = 0$$

$$x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0$$

$$x = -8 \text{ or } x = 2$$

(3)

10. $f(x) = kx + 7$

$g(x) = 3x - 2$

Given that $gf(1) = 34$

Find the value of k

$$f(1) = k + 7$$

$$g(k+7) = 3(k+7) - 2$$

$$= 3k + 21 - 2$$

$$= 3k + 19$$

$$3k + 19 = 34$$

$$3k = 15$$

$$k = 5$$

(3)

11. $f(x) = \frac{3}{x+4}$ for all positive values of x

Work out $f(x+2) + f(x+1)$

Give your answer as a single fraction in its simplest form.

$$\frac{3}{(x+2)+4} + \frac{3}{(x+1)+4} = \frac{6x+33}{(x+6)(x+5)}$$

$$= \frac{3}{x+6} + \frac{3}{x+5}$$

$$\frac{3(x+5) + 3(x+6)}{(x+6)(x+5)}$$

$$\frac{3x+15+3x+18}{(x+6)(x+5)}$$

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(5)

12. $f(x) = \frac{3-x^2}{8}$ for all values of x

Solve $f(10x) = -7$

$$\frac{3-(10x)^2}{8} = -7$$

$$3-100x^2 = -56$$

$$100x^2 = 59$$

$$x^2 = \frac{59}{100}$$

$$x = \pm \frac{\sqrt{59}}{10}$$

$$x = -\frac{\sqrt{59}}{10} \text{ or } x = \frac{\sqrt{59}}{10}$$

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(4)

13. $f(x) = x^2 + 5$

$$g(x) = x - 8$$

Solve $fg(x) = gf(x)$

$$(x-8)^2 + 5 = x^2 + 5 - 8$$

$$x^2 - 16x + 64 + 5 = x^2 - 3$$

$$x^2 - 16x + 69 = x^2 - 3$$

$$-16x + 69 = -3$$

$$-16x = -72$$

$$x = \frac{9}{2}$$

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(5)

14. $f(x) = x^2 - 2x$

Solve $f(3x) - f(x - 1) = 4$

Give your answers to 2 decimal places

$$(3x)^2 - 2(3x) - \left((x-1)^2 - 2(x-1) \right) = 4$$

$$9x^2 - 6x - (x^2 - 2x + 1 - 2x + 2) = 4$$

$$9x^2 - 6x - (x^2 - 4x + 3) = 4$$

$$8x^2 - 2x - 3 = 4$$

$$8x^2 - 2x - 7 = 0$$

$$a = 8 \quad b = -2 \quad c = -7$$

$$x = 1.07 \quad \text{or} \quad x = -0.82$$

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(7)