Name:

Level 2 Further Maths

Factor Theorem

Corbettmaths

Ensure you have: Pencil or pen, a calculator

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. Use factor theorem to show that $(x - 1)$ is a factor of $x^3 - 3x^2 - 13x + 15$

$f(1) = 1^3 - 3(1)^2 - 13(1) + 15$

$f(1) = 1 - 3 - 13 + 15 = 0$

$\therefore (x-1)$ is a factor of $x^3 - 3x^2 - 13x + 15$

(1)

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2. Use factor theorem to show that $(x - 3)$ is a factor of $x^3 - 10x^2 + 21x$

$f(3) = 3^3 - 10 \times 3^2 + 21 \times 3$

$f(3) = 27 - 90 + 63$

$f(3) = 0$

$\therefore (x-3)$ is a factor of $x^3 - 10x^2 + 21x$

(1)

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3. Use factor theorem to show that $(x + 4)$ is a factor of $x^3 + 4x^2 - 3x - 12$

$f(-4) = (-4)^3 + 4(-4)^2 - 3(-4) - 12$

$f(-4) = -64 + 64 + 12 - 12 = 0$

$f(-4) = 0$

$\therefore (x+4)$ is a factor of $x^3 + 4x^2 - 3x - 12$

(1)

4. Use factor theorem to show that $(2x-1)$ is a factor of $2x^3+7x^2+2x-3$

$f(\tfrac{1}{2}) = 2(\tfrac{1}{2})^3 + 7(\tfrac{1}{2})^2 + 2(\tfrac{1}{2}) - 3$

$f(\tfrac{1}{2}) = \tfrac{1}{4} + \tfrac{7}{4} + 1 - 3$

$f(\tfrac{1}{2}) = 2 + 1 - 3 = 0$

$\therefore$ $(2x-1)$ is a factor of $2x^3+7x^2+2x-3$

(2)

5. $f(x) = 4x^3+5x^2-23x-6$

Use the factor theorem to show that $(4x+1)$ is a factor of $f(x)$

$f(-\tfrac{1}{4}) = 4(-\tfrac{1}{4})^3 + 5(-\tfrac{1}{4})^2 - 23(-\tfrac{1}{4}) - 6$

$f(-\tfrac{1}{4}) = -\tfrac{1}{16} + \tfrac{5}{16} + \tfrac{23}{4} - 6$

$f(-\tfrac{1}{4}) = 0$

$\therefore$ $(4x+1)$ is a factor of $4x^3+5x^2-23x-6$

(2)

6. Use the factor theorem to show that $(x+5)$
is **not** a factor of $x^3-12x^2+47x-35$

$f(-5) = (-5)^3 - 12(-5)^2 + 47(-5) - 35$

$f(-5) = -125 - 300 - 235 - 35$

$f(-5) = -695$

$\therefore$ $(x+5)$ is not a factor of $x^3-12x^2+47x-35$

(2)

7. (a) Use the factor theorem to show that $(x-1)$ is a factor of $x^3 - x^2 - 4x + 4$

$f(1) = 1^3 - 1^2 - 4 + 4$

$f(1) = 1 - 1 - 4 + 4 = 0$

$\therefore$ $(x-1)$ is a factor of $x^3 - x^2 - 4x + 4$

(1)

(b) Hence, factorise fully $x^3 - x^2 - 4x + 4$

$$
\begin{array}{r|l}
 & x^2 \quad\quad -4 \\
\hline
x-1 & x^3 - x^2 - 4x + 4 \\
 & x^3 - x^2 \\
\hline
 & \quad\quad -4x + 4 \\
 & \quad\quad -4x + 4 \\
\hline
 & \quad\quad\quad 0
\end{array}
$$

$(x-1)(x^2-4)$

$(x-1)(x-2)(x+2)$

....

(3)

8. (a) Use the factor theorem to show that $(x-2)$ is a factor of $x^3 - 9x^2 + 20x - 12$

$f(2) = 2^3 - 9(2)^2 + 20(2) - 12$

$f(2) = 8 - 36 + 40 - 12$

$f(2) = 0 \quad \therefore (x-2)$ is a factor of $x^3 - 9x^2 + 20x - 12$

(1)

(b) Hence, factorise fully $x^3 - 9x^2 + 20x - 12$

$$\begin{array}{r l}
 & x^2 - 7x + 6 \\
x-2 & \overline{)\, x^3 - 9x^2 + 20x - 12} \\
 & x^3 - 2x^2 \\
\hline
 & -7x^2 + 20x - 12 \\
 & -7x^2 + 14x \\
\hline
 & 6x - 12 \\
 & 6x - 12 \\
\hline
 & 0
\end{array}$$

$(x-2)(x^2 - 7x + 6)$

$(x-2)(x-6)(x-1)$

$(x-6)(x-2)(x-1)$

(3)

9. (a) Use the factor theorem to show that $(x + 4)$ is a factor of $2x^3 + 5x^2 - 14x - 8$

$f(-4) = 2(-4)^3 + 5(-4)^2 - 14(-4) - 8$

$f(-4) = -128 + 80 + 56 - 8 = 0$

$\therefore (x+4)$ is a factor of $2x^3 + 5x^2 - 14x - 8$

(1)

(b) Hence, factorise fully $2x^3 + 5x^2 - 14x - 8$

$$
\begin{array}{r|l}
 & 2x^2 - 3x - 2 \\
\hline
x+4 & 2x^3 + 5x^2 - 14x - 8 \\
 & 2x^3 + 8x^2 \\
\hline
 & \quad -3x^2 - 14x - 8 \\
 & \quad -3x^2 - 12x \\
\hline
 & \qquad -2x - 8 \\
 & \qquad -2x - 8 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$(x+4)(2x^2 - 3x - 2)$

$(x+4)(2x+1)(x-2)$

....

(4)

10. (a) Use the factor theorem to show that $(2x - 3)$ is a factor of $2x^3 + x^2 - 12x + 9$

$f\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 + \left(\frac{3}{2}\right)^2 - 12\left(\frac{3}{2}\right) + 9$

$f\left(\frac{3}{2}\right) = \frac{27}{4} + \frac{9}{4} - 18 + 9$

$= 9 - 18 + 9 = 0$

$\therefore (2x-3)$ is a factor of $2x^3 + x^2 - 12x + 9$

(2)

(b) Hence, factorise fully $2x^3 + x^2 - 12x + 9$

$$
\begin{array}{r|l}
 & x^2 + 2x - 3 \\
2x-3 & 2x^3 + x^2 - 12x + 9 \\
 & 2x^3 - 3x^2 \\
 & \quad 4x^2 - 12x + 9 \\
 & \quad 4x^2 - 6x \\
 & \qquad -6x + 9 \\
 & \qquad -6x + 9 \\
 & \qquad\qquad 0
\end{array}
$$

$(2x-3)(x^2+2x-3)$

$(2x-3)(x+3)(x-1)$

$(2x-3)(x+3)(x-1)$

(3)

11. (a) Use the factor theorem to show that $(x-2)$ and $(x+5)$ are factors of $x^3+2x^2-13x+10$

$f(2) = 8 + 8 - 26 + 10$

$f(2) = 0$

$\therefore (x-2)$ is a factor of $x^3+2x^2-13x+10$

$f(-5) = -125 + 50 + 65 + 10 = 0$

$\therefore (x+5)$ is also a factor

(2)

(b) Use the factor theorem to show that $(x-2)$ and $(x+5)$ are also factors of $x^3+11x^2+14x-80$

$f(2) = 8 + 44 + 28 - 80 = 0$

$f(-5) = -125 + 275 - 70 - 80 = 0$

$\therefore$ both $(x-2)$ & $(x+5)$ are factors of $x^3+11x^2+14x-80$

(2)

(c) Hence, simplify fully $\dfrac{x^3+2x^2-13x+10}{x^3+11x^2+14x-80}$

$$\begin{array}{r} x^2+4x-5 \\ x-2\,\overline{)\,x^3+2x^2-13x+10} \\ \underline{x^3-2x^2} \\ 4x^2-13x+10 \\ \underline{4x^2-8x} \\ -5x+10 \end{array}$$

$$\begin{array}{r} x^2+13x+40 \\ x-2\,\overline{)\,x^3+11x^2+14x-80} \\ \underline{x^3-2x^2} \\ 13x^2+14x-80 \\ \underline{13x^2-26x} \\ 40x-80 \\ \underline{40x-80} \\ 0 \end{array}$$

$(x-2)(x+5)(x-1)$

$(x-2)(x+5)(x+8)$

$\dfrac{x-1}{x+8}$

(3)

12. (a) Show that $(x + 3)$ is a factor of $x^3 + 3x^2 - 49x - 147$

$f(-3) = -27 + 27 + 147 - 147$

$f(-3) = 0$

$\therefore$ $(x+3)$ is a factor of $x^3 + 3x^2 - 49x - 147$

(2)

(b) Hence, or otherwise, find all the solutions of $x^3 + 3x^2 - 49x - 147 = 0$

$$\begin{array}{r|l} & x^2 \quad\quad - 49 \\ \hline x+3 & x^3 + 3x^2 - 49x - 147 \\ & x^3 + 3x^2 \\ \hline & \quad\quad\quad - \cancel{4}9x - 147 \\ & \quad\quad\quad -49x - 147 \\ \hline & \quad\quad\quad\quad\quad 0 \end{array}$$

$(x+3)(x^2-49)$

$(x+3)(x-7)(x+7) = 0$

$x = -3$, $x = 7$, $x = -7$

(4)

13. Factorise fully $x^3 - 6x^2 + 11x - 6$

$f(1) = 1^3 - 6\times 1 + 11 - 6$

$f(1) = 0$

$\therefore \ (x-1)$ is a factor of $x^3 - 6x^2 + 11x - 6$

$$
\begin{array}{r|l}
 & x^2 - 5x + 6 \\
x-1 & x^3 - 6x^2 + 11x - 6 \\
 & x^3 - x^2 \\
\hline
 & -5x^2 + 11x - 6 \\
 & -5x^2 + 5x \\
\hline
 & 6x - 6 \\
 & 6x - 6 \\
\hline
 & 0
\end{array}
$$

$(x-1)(x^2-5x+6)$

$(x-1)(x-2)(x-3)$

(5)

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14. $(x - 5)$ is a factor of $x^3 - x^2 - 32x + a$

Work out the value of a

$f(5) = 125 - 25 - 160 + a = 0$ (since factor)

$a = 60$

$a =$ ......60..................

(2)

14. $(x+4)$ is a factor of $x^3+11x^2+ax-72$

Work out the value of a

$f(-4) = 0$ since factor

$-64 + 176 - 4a - 72 = 0$

$40 - 4a = 0$

$4a = 40$

$a = 10$

$a =$ ......10......

(3)

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15. Given $(x-1)$ is a factor of $3x^3-15x^2+ax+a$

Find the value of a

$f(1) = 0$ since factor.

$3 - 15 + a + a = 0$

$-12 + 2a = 0$

$2a = 12$

$a = 6$

$a =$ ......6......

(4)

16. $(x + a)$ is a factor of $x^3 - 7x^2 + ax + 20a$

(a) Show that $a = 2$

~~[illegible]~~

$(x+2) \quad x^3 - 7x^2 + 2x + 40$

$f(-2) = -8 - 28 - 4 + 40 = 0$

$\therefore$

$a = 2$

(2)

(b) Solve $x^3 - 7x^2 + 2x + 40 = 0$

$$\begin{array}{r l}
 & x^2 - 9x + 20 \\
x+2 & \overline{)\, x^3 - 7x^2 + 2x + 40} \\
 & x^3 + 2x^2 \\
 & \overline{\quad -9x^2 + 2x + 40} \\
 & \quad -9x^2 - 18x \\
 & \overline{\qquad 20x + 40} \\
 & \qquad 20x + 40 \\
 & \overline{\qquad\qquad 0}
\end{array}$$

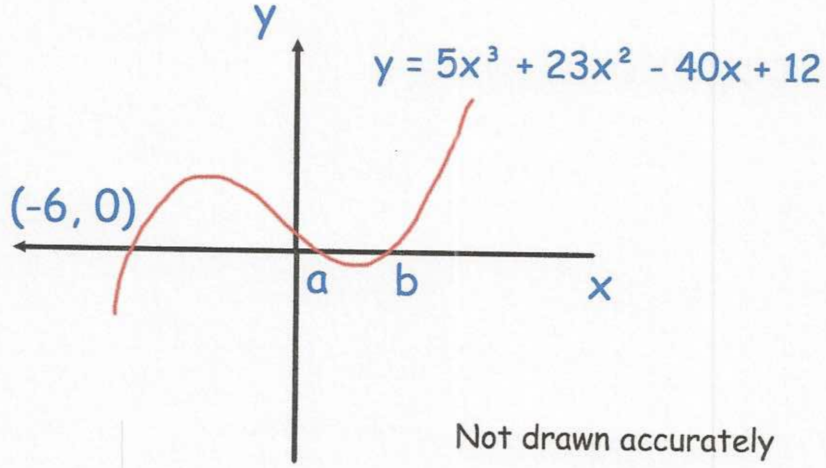
$(x+2)(x^2 - 9x + 20)$

$(x+2)(x-4)(x-5) = 0$

$x = -2,\ x = 4$ or $x = 5$

(4)

17. Below is the graph of $y = 5x^3 + 23x^2 - 40x + 12$



Find the coordinates of the points *a* and *b*, where the graph of $y = 5x^3 + 23x^2 - 40x + 12$ crosses the x-axis.

($(x+6)$ is a factor)

$$
\begin{array}{r}
5x^2 - 7x + 2 \\
x+6 \overline{) 5x^3 + 23x^2 - 40x + 12} \\
\underline{5x^3 + 30x^2} \\
-7x^2 - 40x + 12 \\
\underline{-7x^2 - 42x} \\
2x + 12 \\
\underline{2x + 12} \\
0
\end{array}
$$

$a = \left(\frac{2}{5}, 0\right)$ ....

$b = (1, 0)$

(4)

$(x+6)(5x^2 - 7x + 2)$

$(x+6)(5x-2)(x-1)$

$5x = 2$

$x = \frac{2}{5}$ $x = 1$

18. Solve $x^3 - 19x^2 + 103x - 165 = 0$

$f(-3) = -27 - 171 - 309 - 165 \neq 0$

$f(3) = 27 - 171 + 309 - 165 = 0$

$\therefore (x-3)$ is a factor

$$
\begin{array}{r|l}
 & x^2 - 16x + 55 \\
\hline
x-3 & x^3 - 19x^2 + 103x - 165 \\
 & x^3 - 3x^2 \\
\hline
 & -16x^2 + 103x - 165 \\
 & -16x^2 + 48x \\
\hline
 & 55x - 165 \\
 & 55x - 165 \\
\hline
 & 0
\end{array}
$$

$(x-3)(x^2 - 16x + 55)$

$(x-3)(x-5)(x-11) = 0$

$x = 3$, $x = 5$ or $x = 11$

(5)