LEVEL 2 CERTIFICATE
FURTHER MATHEMATICS
8360/2

Paper 2 Calculator

Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between *a* and *b* inclusive.
3.14…	Accept answers which begin 3.14 eg 3.14, 3.142, 3.1416

Examiners should consistently apply the following principles.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1(a)	**Alternative method 1**		
	$3a = 4(2a + 3)$ or $3a = 8a + 12$ or $5a = 4b$	M1	oe equation
	$(a =)\ -\frac{12}{5}$ or $-2\frac{2}{5}$ or -2.4	A1	
	$(b =)\ -3$	A1ft	ft their $a \times 1.25$ evaluated or $\frac{\text{their } 5a}{4}$ evaluated $a \neq 0$ if awarding ft M1 is implied
	Alternative method 2		
	$3a + 5a = 4(2a + 3) + 4b$ or $8a = 8a + 12 + 4b$ or $4b = -12$	M1	oe equation
	$(b =)\ -3$	A1	
	$(a =)\ -\frac{12}{5}$ or $-2\frac{2}{5}$ or -2.4	A1ft	ft their $b \times 0.8$ evaluated or $\frac{\text{their } 4b}{5}$ evaluated $b \neq 0$ if awarding ft M1 is implied
	Additional Guidance		
	Alt 1 $a = 2$ $b = 2.5$ ($5a = 4b$ is implied)		M1A0A1ft
	$\begin{pmatrix} 3a \\ 5a \end{pmatrix} = \begin{pmatrix} 8a+12 \\ 4b \end{pmatrix}$		M0
	Accept $\frac{-12}{5}$ or $\frac{12}{-5}$ for $-\frac{12}{5}$ (apply throughout scheme for values)		
	Only solutions seen with one correct and the other incorrect (or missing)		2 marks

Q	Answer	Mark	Comments
1(b)	$2m + 2 = 1$ or $2m + 1 = 0$ or $\frac{1-2}{2}$ or $\begin{pmatrix} 2m+2 & 2m+1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	M1	oe equation or calculation
	$-\frac{1}{2}$ or -0.5	A1	
	Additional Guidance		
	Condone missing brackets in $\begin{pmatrix} 2m+2 & 2m+1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		
	Allow $\begin{pmatrix} 2m+2 & 2m+1 \\ 2-2 & 2-1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$		
	Mark positively eg error in matrix multiplication but $2m + 2 = 1$ and answer –0.5		M1A1
	More than one answer given is A0 eg $m + 2 = 1$ and $2m + 1 = 0$ (mark positively) Answer –1 and –0.5		M1 A0

Q	Answer	Mark	Comments
2	$\left(\frac{4+6}{2}, \frac{1+9}{2}\right)$ or (5, 6)	M1	oe eg $\left(4+\frac{6-4}{2}, 1+\frac{11-1}{2}\right)$ may be on diagram
	$\frac{1--3}{4-10}$ or $\frac{4}{-6}$ or $\frac{0-\text{their } 6}{14-\text{their } 5}$ or $\frac{-6}{9}$	M1	oe method for at least one gradient or at least one unsimplified gradient seen eg $\frac{-3-1}{10-4}$ or $\frac{-4}{6}$ or $\frac{\text{their } 6-0}{\text{their } 5-14}$ or $\frac{6}{-9}$ $\frac{6-0}{5-14}$ or $\frac{6}{-9}$ is M1M1
	$\frac{1--3}{4-10}$ or $\frac{4}{-6}$ and $\frac{0-6}{14-5}$ or $\frac{-6}{9}$ and shows that the gradients are equal	A1	oe method for both gradients or two unsimplified gradients seen and gradients shown to be equal eg $\frac{4}{-6}$ and $\frac{-6}{9}$ and these are both $-\frac{2}{3}$ SC2 (5, 6) and at least one gradient given as $-\frac{2}{3}$ SC1 at least one gradient given as $-\frac{2}{3}$

Additional Guidance is on the next page

Q	Answer	Mark	Comments

Q	Additional Guidance	
2 cont	Mark intention for 1st M1 eg condone 5, 6	M1
	$\frac{4}{-6} = -\frac{2}{3}$ and $\frac{-6}{9} = -\frac{2}{3}$	M2A1
	$\frac{1--3}{4-10} = -\frac{2}{3}$ and $\frac{0-6}{14-5} = -\frac{2}{3}$	M2A1
	$\frac{4}{-6} = \frac{-6}{9}$	M2A1
	$\frac{4}{-6}$ and $\frac{-6}{9}$ and parallel	M2A0
	$\frac{4}{6}$ is 2nd M0 unless recovered to $-\frac{4}{6}$	
	$\frac{4}{6}$ recovered to $-\frac{4}{6}$ and $\frac{6}{9}$ recovered to $-\frac{6}{9}$ could go on to score full marks	
	both gradients = $-\frac{2}{3}$ with no method or unsimplified gradients seen cannot score the A mark	
	$\frac{4}{-6}x$ or $\frac{-6}{9}x$ do not score 2nd M1 unless recovered	
	Equation of a line does not score 2nd M1 unless a method or unsimplified gradient seen	
	Using the reciprocals of gradients can score a maximum of M1M0A0	
	Allow –0.66… or –0.67 for $-\frac{2}{3}$ and $\frac{4}{-6}$ etc Ignore conversion attempt after a correct fraction is seen	
	or method for $\frac{4}{-6}$ $1 = 4m + c$ and $-3 = 10m + c$ $4 = -6m$ $\frac{4}{-6} = m$ (similar method possible for $\frac{-6}{9}$)	(2nd) M1

Q	Answer				Mark	Comments
3	$a^2 < 0$	☐	☐	✓	B4	B1 for each correct row
	$-1 < b^3 < 1$	✓	☐	☐		
	$\frac{b}{a} < 0$	☐	✓	☐		
	$a - b > 0$	☐	✓	☐		

Additional Guidance

Guidance	Mark
Two boxes ticked in a row with other 3 rows fully correct	B3
One row correct, two rows blank, all three boxes ticked in another row	B1
Only crosses used instead of ticks eg cross in all 4 correct boxes with all other boxes blank	B4
Ticks and crosses used – only mark the ticks for that row eg Top row has X X ✓ scores B1 for that row Second row has X ✓ X scores B0 for that row	

Q	Answer	Mark	Comments
4	$(y =)\ \frac{3}{2}x\ldots$ or $(y =)\ 1.5x\ldots$ or $\frac{3}{2}$ or 1.5	M1	oe eg $(y =)\ \frac{3x-9}{2}$
	$\frac{x^5-17}{10} = \frac{3}{2}$	M1dep	oe implies M2
	$x^5 = \frac{3}{2} \times 10 + 17$ or $\sqrt[5]{32}$ or correctly rearranges $\frac{x^5-17}{10} = k$ to the form $x^5 =$ (k any non-zero value)	M1	oe eg $x^5 = 15 + 17$ or $x^5 = 32$ or $\sqrt[5]{15+17}$ must rearrange to the form $x^5 =$
	2	A1	

Additional Guidance is on the next page

Q	Answer	Mark	Comments

Q	Additional Guidance	
4 cont	Condone error seen in rearrangement of $3x - 2y = 9$ if gradient is $\frac{3}{2}$ May go on to score M3A1	
	$\frac{x^5 - 17}{10} = \frac{3}{2}x$	M1M0M0A0
	(gradient =) 3 $\frac{x^5 - 17}{10} = 3$ $x^5 = 30 + 17$ (3rd M is not dependent) 2.16	M0M0dep M1 A0
	$\frac{3}{2}$ $\frac{x^5 - 17}{10} = -\frac{2}{3}$ $x^5 = -\frac{2}{3} \times 10 + 17$ (3rd M is not dependent) 1.595	M1 M0 M1 A0
	Condone answer (2, …)	
	2 embedded	M3A0

Q	Answer	Mark	Comments
5(a)	a^{-2}	B2	B1 applies an index law or changes fourth root to fractional or decimal power in a correct expression eg $\sqrt[4]{a^{-8}}$ or $(a^{-8})^{\frac{1}{4}}$ or $(a^{8})^{-\frac{1}{4}}$ or $a^{-\frac{8}{4}}$ or $\frac{1}{a^{\frac{8}{4}}}$ or $\frac{1}{a^2}$ or $a^{\frac{1}{4}} \times a^{-\frac{9}{4}}$ or $a^{\frac{1}{4}} \times \frac{1}{a^{\frac{9}{4}}}$ or $\left(\frac{1}{a^8}\right)^{\frac{1}{4}}$ or $\sqrt[4]{\frac{1}{a^8}}$ or $\sqrt[4]{a \times \frac{1}{a^9}}$ or $\left(a \times a^{-9}\right)^{\frac{1}{4}}$ or $\left(a \times \frac{1}{a^9}\right)^{\frac{1}{4}}$

Additional Guidance

$a^{\frac{-8}{4}}$ or $a^{\frac{8}{-4}}$	B1
a^{-2} in working with –2 on answer line	B1
a^{-2} in working with $\frac{1}{a^2}$ on answer line	B1
B1 response followed by further work is still awarded B1	
Allow 0.25 for $\frac{1}{4}$ etc	
Allow recovery of missing brackets	

Q	Answer	Mark	Comments
5(b)	$32c^2d^2$ or $32(cd)^2$	B3	B2 (numerator =) $64c^3d^6$ or single term answer with two of 32, c^2 and d^2 (not in a denominator) B1 single term answer with one of 32, c^2 and d^2 (not in a denominator) SC2 factorised correct expression eg $16cd(2cd)$
	Additional Guidance		
	$2c^2d^2$ or $32c^2d$ or $32c^2$ or $\frac{32d^2}{c^3}$ or $\frac{c^2d^2}{32}$ or $64(cd)^2$ etc		B2
	$32c^3d$ or c^2 or $\frac{d^2}{c}$ or $\frac{c^2d}{32}$ or $\frac{32}{c^2}$ etc		B1
	$\frac{32c^2d^2}{1}$ or $\frac{32(cd)^2}{1}$		B2
	Allow denominator of 1 in a B2 or B1 answer eg $\frac{32c^2d}{1}$		B2
	Multiplication signs in a correct expression eg $32 \times c^2 \times d^2$		B2
	Allow multiplication signs in a B2, SC2 or B1 answer eg $32 \times c^3 \times d$		B1
	Do not accept 2^5 for 32 eg 2^5c^2d		B1
	If answer line scores B1 or B0 check working lines for possible response for up to 2 marks		
	$32c^2d^2$ in working with different answer on answer line		B2

Q	Answer	Mark	Comments
6(a)	$A\ (-\frac{3}{2}, 0)$ and $B\ (2, 0)$	B2	oe B1 $A\ (-\frac{3}{2}, 0)$ oe or $B\ (2, 0)$ SC1 $A\ (2, 0)$ and $B\ (-\frac{3}{2}, 0)$ oe
	Additional Guidance		
	Ignore the diagram		

Q	Answer	Mark	Comments
6(b)	$-\frac{3}{2} < x < 2$ or $2 > x > -\frac{3}{2}$	B1ft	oe correct or ft their values from (a) must be a single inequality in x

Additional Guidance is on the next page

Q	Answer	Mark	Comments

Q	Additional Guidance	
6(b) cont	$-\frac{3}{2} \leqslant x < 2$	B0
	$-\frac{3}{2} > x < 2$	B0
	$-\frac{3}{2} < x$ and $x < 2$	B0
	their (a) A (–2, 0) and B ($\frac{3}{2}$, 0) (B0 in (a)) (b) $-2 < x < \frac{3}{2}$	B1ft
	their (a) A (2, 0) and B ($-\frac{3}{2}$, 0) (SC1 in (a)) (b) $2 < x < -\frac{3}{2}$	B0ft
	their (a) A (–3, 0) and B (2, 0) (B1 in (a)) (b) $2 < x < -3$	B0ft
	their (a) A (4, 0) and B (–2, 0) (B0 in (a)) (b) $-2 < x < 4$	B1ft
	Only one value in (a) can only score in (b) for $-\frac{3}{2} < x < 2$ or $2 > x > -\frac{3}{2}$	

Q	Answer	Mark	Comments
7(a)	Horizontal straight line	B1	mark intention
	Additional Guidance		
	Ignore any attempt at an equation		
	Mark the entire graph on the grid		
	Ignore any graph not on the grid		
	Line clearly drawn on the x-axis		B1
	Line does not need to start from the y-axis		
	Ignore any points plotted		

Q	Answer	Mark	Comments
7(b)	Straight line with gradient > 0	B1	mark intention
	Additional Guidance		
	Ignore any attempt at an equation		
	Mark the entire graph on the grid		
	Ignore any graph not on the grid		
	Vertical line		B0
	A straight line joined to another line with a different gradient		B0
	Line does not need to start at (0, 0)		
	Ignore any points plotted		

Q	Answer	Mark	Comments
8(a)	$7 + 12\sqrt{5} + 6(9 - 2\sqrt{5})$ or $12\sqrt{5} + 6(-2\sqrt{5}) = 0$ or $12\sqrt{5} \div 2\sqrt{5} = 6$ or states that need to add 6 lots of $(9 - 2\sqrt{5})$ or 7th term	M1	oe eg $7 + 6 \times 9$ or $7 + 54$ or $6 \times -2 = -12$ allow $7 + 12\sqrt{5} + (n - 1)(9 - 2\sqrt{5})$ with $n = 7$ allow $7 + 12\sqrt{5} + n(9 - 2\sqrt{5})$ with $n = 6$
	61	A1	
	Additional Guidance		
	61 in working lines with 7(th) on answer line		M1A0
	If repeatedly adding $(9 - 2\sqrt{5})$ they must stop after adding 6 lots or clearly select the relevant one		
	Answer 6 or 6th term with M1 not seen		M0A0
	Ignore any conversions to decimals		
	Beware $(9 - 2\sqrt{5})(9 + 2\sqrt{5}) = 61$		M0A0

Q	Answer	Mark	Comments
8(b)	$\frac{29}{5}$ or $5\frac{4}{5}$ or 5.8	B2	oe eg $5\frac{8}{10}$ B1 any two of 1, $\frac{11}{5}$, $\frac{26}{10}$ oe values

Additional Guidance

Terms must be evaluated for B1 unless correct answer seen eg1 $\frac{3-1}{1+1}$ + $\frac{12-1}{4+1}$ + $\frac{27-1}{9+1}$ eg2 $\frac{3-1}{1+1}$ + $\frac{12-1}{4+1}$ + $\frac{27-1}{9+1}$ = 5.8	 B0 B2
1 7 2.6	B1
Ignore conversion attempts after a correct value seen	

Q	Answer	Mark	Comments
8(c)	**Alternative method 1**		
	(Second differences =) 4 or $2n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	–3 – 2 3 – 8 (13 – 18 27 – 32) or –5 –5 (–5 –5)	M1dep	subtracts $2n^2$ from the given terms
	$2n^2 - 5$	A1	oe eg $2n^2 + 0n - 5$ does not need terms collected
	Alternative method 2		
	(Second differences =) 4 or $2n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	$3a + b = 3 - -3$ and substitutes $a = 2$ or $b = 0$	M1dep	oe
	$2n^2 - 5$	A1	oe eg $2n^2 + 0n - 5$ does not need terms collected

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Q	Answer	Mark	Comments
8(c) cont	**Alternative method 3**		
	Any three of $a + b + c = –3$ $4a + 2b + c = 3$ $9a + 3b + c = 13$ $16a + 4b + c = 27$	M1	
	$3a + b = 3 – –3$ and $5a + b = 13 – 3$ or $a = 2$ and $b = 0$	M1dep	oe obtains two correct equations in same two variables from their equations
	$2n^2 – 5$	A1	oe eg $2n^2 + 0n – 5$ does not need terms collected
	Alternative method 4		
	(Second differences =) 4 or $2n^2$	M1	second differences seen at least once and not contradicted may be seen by the sequence
	$2 × 1^2 + b × 1 – 5 = –3$ or $2 + b – 5 = –3$ or $b = 0$	M1dep	$2n^2 + bn – 5 = –3$ with $n = 1$ substituted oe eg $2 × 2^2 + b × 2 – 5 = 3$
	$2n^2 – 5$	A1	oe eg $2n^2 + 0n – 5$ does not need terms collected
	Additional Guidance		
	Condone working in a different variable		
	Alt 1 2nd M1 Subtracting given terms from $2n^2$ leading to 5 5 (5 5) must be recovered eg final answer $2n^2 – 5$ ($2n^2 – 5n$ or $2n^2 – 5n – 5$ is not a recovery)		
	Answer $2n^2$ …… scores at least M1		
	Condone $n = 2n^2 – 5$ or $2n^2 – 5 = 0$		M2A1

Q	Answer	Mark	Comments
9	$(p + 6)^{10}(p + 5)$ or $(p + 5)(p + 6)^{10}$	B2	B1 $(p + 6)^{10}(p + 6 – 1)$ or states $x = p + 6$ and $x^{10}(x – 1)$ (any letter for x other than p) or correct partial factorisation eg $(p + 6)[(p + 6)^{10} – (p + 6)^9]$ or $(p + 6)^2[(p + 6)^9 – (p + 6)^8]$

Additional Guidance

Any shape of bracket may be used	
$(p + 6)^{10}((p + 6) – 1)$	B1
Missing brackets must be recovered eg $p + 5\ (p + 6)^{10}$ not recovered and B1 response not seen	B0
Condone $(p + 6)^{10}(p + 5$	B2
Condone $(p + 6)^{10}(p + 6 – 1$	B1
$(p + 6)^{10}(p + 5)$ followed by expansion attempt	B1
B1 response followed by expansion attempt	B1
$(p + 6)^{10} \times (p + 5)$	B1
Condone multiplication signs for B1 eg $(p + 6)^{10} \times (p + 6 – 1)$	B1
$(p+6)^{11}\left[1-\dfrac{1}{p+6}\right]$	B1

Q	Answer	Mark	Comments
10(a)	$f(x) \leqslant 25$ or $25 \geqslant f(x)$	B2	B1 $f(x) < 25$ or $k \leqslant f(x) \leqslant 25$ or $k < f(x) \leqslant 25$ where k is any number < 25 SC1 $\leqslant 25$ or $x \leqslant 25$

Additional Guidance

Condone $f(x)$ replaced by eg y or f or fx or F(x) or F or Fx or $x^3 - 2$ in B2 or B1 responses	
Equivalent inequalities may be seen $25 > f(x)$	B1
Allow $-\infty < f(x) \leqslant 25$	B2
Condone $-\infty \leqslant f(x) \leqslant 25$	B2
$-\infty < f(x) < 25$ or $-\infty \leqslant f(x) < 25$	B1
$[-\infty, 25]$ or $(-\infty, 25]$	B1
$(-\infty, 25)$	B0
Condone $f(x) = \leqslant 25$	B2
Condone $f(x) = < 25$	B1
Condone $f(x) = x \leqslant 25$	SC1
$f(x) \leqslant 25$ in working with list of integers on answer line	B1
Only a list of integers	B0

Q	Answer	Mark	Comments
10(b)	$1 \leqslant g(x) \leqslant 5$ or $5 \geqslant g(x) \geqslant 1$	B2	B1 $1 \leqslant g(x) < 5$ or $1 < g(x) \leqslant 5$ or $1 < g(x) < 5$ or $g(x) \geqslant 1$ and $g(x) \leqslant 5$ or $1 \leqslant g(x) \leqslant k$ where k is a constant > 1 or $p \leqslant g(x) \leqslant 5$ where p is a constant < 5 SC1 $1 \leqslant x \leqslant 5$

Additional Guidance

Condone g(x) replaced by eg y or g or gx or f(x) or f or fx or $5 - x^2$ in B2 or B1 responses	
Equivalent inequalities may be seen eg $5 \geqslant g(x) > 1$	B1
Only $g(x) \geqslant 1$ given as the answer	B0
Only $g(x) \leqslant 5$ given as the answer	B0
$1 \leqslant g(x) \leqslant 4$	B1
$1 \leqslant g(x) < 4$	B0
$0 \leqslant g(x) \leqslant 5$	B1
$0 < g(x) \leqslant 5$	B0
Invalid statements do not score eg1 $1 \leqslant g(x) \geqslant 5$ eg2 $1 \geqslant g(x) \leqslant 5$ eg3 $6 \leqslant g(x) \leqslant 5$	 B0 B0 B0
[1, 5]	B1
[1, 5) or (1, 5] or (1, 5) or 1 – 5 or 5 – 1	B0
$1 \leqslant g(x) \leqslant 5$ in working with list of integers on answer line	B1
Only a list of integers	B0

Q	Answer	Mark	Comments
11	$x = -2$	B1	
	Additional Guidance		

Q	Answer	Mark	Comments
12	**Alternative method 1**		
	$\frac{1}{2} \times \frac{4}{3} \times \pi \times (6a)^3$ or $\frac{2}{3} \times \pi \times 216a^3$ or $144\pi a^3$	M1	oe eg $\frac{1}{2} \times \frac{4}{3} \times \pi \times \left(\frac{12a}{2}\right)^3$ or $\frac{2}{3} \times \pi \times (6a)^3$
	$a^3 = \frac{486\pi}{144\pi}$ or $a^3 = \frac{27}{8}$ or $a^3 = 486 \div \left(\frac{2}{3} \times 6^3\right)$ or $a^3 = 3.375$ or $\sqrt[3]{3.375}$	A1	oe equation of form $a^3 =$ or calculation allow $(6a)^3 = 729$ or $6a = 9$
	$\frac{3}{2}$ or $1\frac{1}{2}$ or 1.5	A1	SC1 answer 0.75 oe or answer 1.19… or answer 4.95…

Mark scheme and Additional Guidance continue on the next page

Q	Answer	Mark	Comments
12 cont	**Alternative method 2**		
	$r^3 = \dfrac{486\pi}{\frac{2}{3}\pi}$ or $r^3 = 729$ or $\sqrt[3]{729}$ or 9	M1	oe equation of form $r^3 =$ or calculation
	$6a = \sqrt[3]{\dfrac{486\pi}{\frac{2}{3}\pi}}$ or $6a = 9$ or $9 \div 6$	A1	oe equation or calculation allow $(6a)^3 = 729$
	$\dfrac{3}{2}$ or $1\dfrac{1}{2}$ or 1.5	A1	SC1 answer 0.75 oe or answer 1.19… or answer 4.95…
	Additional Guidance		
	Allow recovery of missing brackets		
	Allow use of π = [3.14, 3.142]		

Q	Answer	Mark	Comments
13	$x(1 - x^2)$ or $2x(1 + x)$ or $x(2 + 2x)$ or $\dfrac{1-x^2}{2+2x}$	M1	implied by 2nd M1 oe factorisation eg $-x(x^2 - 1)$
	$x(1 + x)(1 - x)$ or $\dfrac{x(1-x^2)}{2x(1+x)}$ or $\dfrac{1-x^2}{2(1+x)}$ or $\dfrac{(1+x)(1-x)}{2+2x}$	M1dep	implies M2 oe factorisation eg $-x(x + 1)(x - 1)$
	$\dfrac{x(1+x)(1-x)}{2x(1+x)}$ or $\dfrac{(1+x)(1-x)}{2(1+x)}$ or $\dfrac{x(1-x)}{2x}$	M1dep	implies M3 oe factorisation eg $\dfrac{-x(x+1)(x-1)}{2x(1+x)}$
	$\dfrac{1-x}{2}$ with M3 seen	A1	oe simplest form eg $\dfrac{1}{2}(1 - x)$ or $\dfrac{1}{2} - \dfrac{1}{2}x$ or $\dfrac{-x+1}{2}$
	Additional Guidance		
	$\dfrac{x(1+x)(1-x)}{2x(1+x)}$ or $\dfrac{(1+x)(1-x)}{2(1+x)}$ or $\dfrac{x(1-x)}{2x}$ is sufficient working		M3
	$2(x + x^2)$ with no further work		M0
	$\dfrac{x-1}{-2}$ with M3 seen or $-\dfrac{1}{2}(x - 1)$ with M3 seen or $\dfrac{-(x-1)}{2}$ with M3 seen		M3A1

Q	Answer	Mark	Comments
14	$a^2 + (3a)^2 - 2 \times a \times 3a \times \cos 120$ or $\cos 120 = \dfrac{a^2 + (3a)^2 - b^2}{2 \times a \times 3a}$	M1	oe eg may substitute cos 120 = –0.5 may be seen in a square root
	$b^2 = a^2 + 9a^2 + 3a^2$ or $b^2 = 13a^2$ or $-b^2 = -13a^2$ or $b = \sqrt{13}\,a$	A1	oe equation of the form $b^2 =$ or $b =$ with brackets expanded and terms fully simplified cos 120 = –0.5 substituted
	13 : 1	A1	SC1 7 : 1
	Additional Guidance		
	Allow recovery of missing brackets		
	$a^2 = a^2 + (3a)^2 - 2 \times a \times 3a \times \cos 120$ not recovered		M1M0A0
	$b^2 = 10a^2 - -3a^2$		M1A0A0
	$b^2 = 10a^2 + 3a^2$		M1A1A0

Q	Answer	Mark	Comments
15	**Alternative method 1**		
	$3mp = 3(2p + 1) + p + 5$ or $(m =)\ \frac{3(2p+1)}{3p} + \frac{p+5}{3p}$ or $(m =)\ \frac{6p+3+p+5}{3p}$	M1	oe fractions eliminated or common denominator eg $(m =)\ \frac{3p(2p+1)}{3p^2} + \frac{p(p+5)}{3p^2}$ or $(m =)\ \frac{6p^2+3p+p^2+5p}{3p^2}$
	$3mp = 6p + 3 + p + 5$ or $3mp = 7p + 8$	M1dep	oe brackets expanded and fractions eliminated eg $3mp^2 = 7p^2 + 8p$ implies M2
	$3mp – 7p = 8$ or $\frac{8}{3m-7}$ or $\frac{-8}{7-3m}$	M1dep	oe terms collected eg $p(3m – 7) = 8$ or $7p – 3mp = –8$ implies M3
	$p = \frac{8}{3m-7}$ or $p = \frac{-8}{7-3m}$	A1	oe eg $\frac{8}{3m-7} = p$

Mark scheme and Additional Guidance continue on the next page

Q	Answer	Mark	Comments
15 cont	**Alternative method 2**		
	$(m =)\ \frac{3(2p+1)}{3p} + \frac{p+5}{3p}$ or $(m =)\ \frac{6p+3+p+5}{3p}$	M1	oe common denominator eg $(m =)\ \frac{3p(2p+1)}{3p^2} + \frac{p(p+5)}{3p^2}$ or $(m =)\ \frac{6p^2+3p+p^2+5p}{3p^2}$
	$m = \frac{7p+8}{3p}$ and $m = \frac{7}{3} + \frac{8}{3p}$ and $m - \frac{7}{3} = \frac{8}{3p}$	M1dep	simplifies numerator and isolates term in p eg $m = \frac{7p^2+8p}{3p^2}$ and $m = \frac{7}{3} + \frac{8}{3p}$ and $m - \frac{7}{3} = \frac{8}{3p}$ implies M2
	$\frac{3m-7}{3} = \frac{8}{3p}$	M1dep	converts $m - \frac{7}{3}$ to a single fraction implies M3
	$p = \frac{8}{3m-7}$ or $p = \frac{-8}{7-3m}$	A1	oe eg $\frac{8}{3m-7} = p$
	Additional Guidance		
	$p = \frac{8}{3m-7}$ in working but $\frac{8}{3m-7}$ on answer line		M3A1
	Allow recovery of missing brackets		
	$p = \frac{8}{3m-7}$ followed by incorrect further work		M3A0
	Allow equivalences for A1 eg $p = \frac{\frac{8}{3}}{\frac{3m-7}{3}}$		M3A1
	Do not regard eg $3m(p) = 7p + 8$ as having unexpanded brackets		M1M1dep

Q	Answer	Mark	Comments
16	$\frac{8-5}{2} = \sqrt{1-a}$ or $\frac{3}{2} = \sqrt{1-a}$ or $3^2 = 2^2(1-a)$ or $9 = 4(1-a)$	M1	
	$1-a = \left(\frac{3}{2}\right)^2$ or $1-a = \frac{9}{4}$ or $9 = 4 - 4a$ or $\frac{4-9}{4}$	M1dep	oe equation or calculation eg $1-a = \left(\frac{8-5}{2}\right)^2$ or $1-a = 2.25$ or $\frac{9-4}{-4}$ implies M2
	$-\frac{5}{4}$ or -1.25 or $-1\frac{1}{4}$	A1	
	Additional Guidance		
	$3 = 2\sqrt{1-a}$		M0
	Allow recovery of missing brackets		

Q	Answer	Mark	Comments
17	$x^2 + 3x + x + 3$ with three terms correct or $x^2 + 4x + k$ where k is a non-zero constant	M1	oe expansion attempt of one pair of brackets eg1 $x^2 + 4x + 3x + 12$ with three terms correct or $x^2 + 7x + k$ where k is a non-zero constant eg2 $x^2 + 4x + x + 4$ with three terms correct or $x^2 + 5x + k$ where k is a non-zero constant
	$x^3 + 3x^2 + x^2 + 3x$ or $x^3 + 4x^2 + 3x$ or $4x^2 + 12x + 4x + 12$ or $4x^2 + 16x + 12$	M1dep	attempt at a full expansion with correct multiplication of their 3 or 4 terms by one of the terms in the remaining bracket oe eg $x^3 + 4x^2 + 3x^2 + 12x$ or $x^3 + 7x^2 + 12x$ or $x^2 + 4x + 3x + 12$ or $x^2 + 7x + 12$ ($x^2 + 7x + 12$ must be from an attempt at a full expansion) or $x^3 + 4x^2 + x^2 + 4x$ or $x^3 + 5x^2 + 4x$ or $3x^2 + 12x + 3x + 12$ or $3x^2 + 15x + 12$
	$x^3 + 8x^2 + 19x + 12$	A1	fully correct expansion allow if terms not collected eg $x^3 + 3x^2 + x^2 + 3x + 4x^2 + 12x + 4x + 12$ or $x^3 + 4x^2 + 3x + 4x^2 + 16x + 12$
	$x^2 + 8x + 12$	A1ft	ft M2A0 full simplification of their $(x^3 + 8x^2 + 19x + 12) - x^3 - 7x^2 - 11x$ their $(x^3 + 8x^2 + 19x + 12)$ must be a cubic
	$x^2 + 8x + 12$ and $(x + 6)(x + 2)$ or $(x + 2)(x + 6)$	A1	oe product of brackets

Additional Guidance is on the next page

Q	Answer	Mark	Comments

Q	Additional Guidance	
17 cont	1st M1 Do not allow omissions or extras eg1 $x^2 + 3x + 3$ eg2 $x^2 + 3x + x + 3 + x^2$	 M0 M0
	For the first 2 marks terms may be seen in a grid	
	If 1st A1 has been awarded with terms not collected, A1ft can still be awarded using their simplified cubic eg $x^3 + 4x^2 + 3x + 4x^2 + 16x + 12$ $= x^3 + 8x^2 + 18x + 12$ $x^3 + 8x^2 + 18x + 12 – x^3 – 7x^2 – 11x$ $= x^2 + 7x + 12$	 M1M1A1 A1ftA0
	First A1 may be seen embedded eg $x^3 + 8x^2 + 19x + 12 – x^3 + 7x^2 – 11x$	 M1M1A1
	If an attempt at the expansion of all three brackets in one go is made it must be fully correct to gain M2A1, otherwise M0M0A0 eg $x^2 + 3x + x + 3 + x^2 + 4x$	 M0M0A0
	Allow recovery of missing brackets when subtracting $x^3 + 7x^2 + 11x$ from their cubic	
	For final A1 allow $x^2 + 8x + 12$ and $a = 6$ $b = 2$ or $x^2 + 8x + 12$ and $a = 2$ $b = 6$	
	Ignore equating to zero and/or any 'solving' of an equation	

Q	Answer	Mark	Comments
18	**Alternative method 1**		
	$x - 5 = \frac{k}{2}$ or $x - 5 = -\frac{k}{2}$ or $2(x - 5) = k$ or $2x - 10 = k$ or $2(x - 5) = -k$ or $2x - 10 = -k$	M1	oe linear equation eg $x - 5 = \sqrt{\frac{k^2}{4}}$ or $x = \frac{k}{2} + 5$ or $\sqrt{4}(x - 5) = \sqrt{k^2}$
	$x - 5 = \frac{k}{2}$ and $x - 5 = -\frac{k}{2}$ or $2(x - 5) = k$ and $2(x - 5) = -k$ or $2x - 10 = k$ and $2x - 10 = -k$	A1	oe eg $x - 5 = \pm\frac{k}{2}$ square root(s) must be processed implied by final A1
	$\frac{k}{2} + 5$ and $-\frac{k}{2} + 5$	A1	oe simplest form eg $\frac{10+k}{2}$ and $\frac{10-k}{2}$ or $\frac{k+10}{2}$ and $\frac{k-10}{-2}$ or $5 \pm 0.5k$
	Alternative method 2		
	$4x^2 - 40x + 100 - k^2$ (= 0)	M1	expands and collects terms
	$\frac{--40 \pm \sqrt{(-40)^2 - 4 \times 4 \times (100 - k^2)}}{2 \times 4}$	A1	oe eg $\frac{40 \pm \sqrt{16k^2}}{8}$ or $\frac{40 \pm 4k}{8}$ implied by final A1
	$\frac{k}{2} + 5$ and $-\frac{k}{2} + 5$	A1	oe simplest form eg $\frac{10+k}{2}$ and $\frac{10-k}{2}$ or $\frac{k+10}{2}$ and $\frac{k-10}{-2}$ or $5 \pm 0.5k$
	Additional Guidance		
	Allow recovery of missing brackets		

Q	Answer	Mark	Comments
19(a)	$\sqrt{(3x)^2 + (4x)^2}$ (= 5x) or $\sqrt{9x^2 + 16x^2}$ (= 5x) or $(3x)^2 + (4x)^2 = (5x)^2$ or 3x, 4x, 5x triangle	B1	may be seen in stages eg $9x^2 + 16x^2 = 25x^2$ and $\sqrt{25x^2}$ (= 5x)

Additional Guidance

Only $\sqrt{25x^2}$ (= 5x) seen	B0
Pythagorean triple 3x, 4x, 5x	B1
Pythagorean triple 3, 4, 5	B0
Missing brackets can **not** be recovered eg1 $\sqrt{3x^2 + 4x^2} = 5x$ eg2 $3x^2 + 4x^2 = 9x^2 + 16x^2 = 25x^2$ and $\sqrt{25x^2}$ (= 5x)	 B0 B0
Incorrect statements are B0 (mark the full response) eg1 $9x^2 + 16x^2 = 25x^2 = \sqrt{25x^2}$ (= 5x) eg2 $9x^2 + 16x^2 = 25x^2$ $\sqrt{25}\,x^2$ (= 5x) eg3 $\sqrt{(3x)^2 + (4x)^2} = 5x$ and $3x + 4x = 5x$	 B0 B0 B0
Only uses values for x	B0

Q	Answer	Mark	Comments
19(b)	**Alternative method 1**		
	$0.5 \times 4x \times 3x$ or $6x^2$	M1	oe may be seen on the diagram
	$(6.5x)^2 - (2.5x)^2$ or $42.25x^2 - 6.25x^2$ or $36x^2$	M1	oe eg $(6.5x)^2 - \left(\frac{5x}{2}\right)^2$
	$\sqrt{\text{their } 36x^2}$ or $6x$	M1dep	dep on 2nd M1 may be seen on the diagram
	$0.5 \times 5x \times$ their $6x$ or $15x^2$	M1dep	oe dep on 2nd and 3rd M1
	$21x^2$	A1	allow $p = 21$ if areas $6x^2$ and $15x^2$ seen
	Alternative method 2		
	$0.5 \times 4x \times 3x$ or $6x^2$	M1	oe may be seen on the diagram
	$\cos ACD = \frac{2.5x}{6.5x}$ or $\cos ACD = \frac{5}{13}$	M1	oe
	$\cos^{-1} \frac{2.5x}{6.5x}$ or 67(.3…) or 67.4	M1dep	oe eg $\cos^{-1} \frac{(6.5x)^2 + (5x)^2 - (6.5x)^2}{2 \times 6.5x \times 5x}$ dep on 2nd M1
	$0.5 \times 5x \times 6.5x \times \sin$ their 67(.3…) or $15x^2$	M1dep	oe dep on 2nd and 3rd M1
	$21x^2$	A1	allow $p = 21$ if areas $6x^2$ and $15x^2$ seen

Mark scheme and Additional Guidance continue on the next page

Q	Answer	Mark	Comments
19(b) cont	**Alternative method 3**		
	$0.5 \times 4x \times 3x$ or $6x^2$	M1	oe may be seen on the diagram
	$(5x)^2 = (6.5x)^2 + (6.5x)^2$ $- 2 \times 6.5x \times 6.5x \times \cos D$	M1	oe
	$\cos^{-1} \dfrac{(6.5x)^2 + (6.5x)^2 - (5x)^2}{2 \times 6.5x \times 6.5x}$ or $\cos^{-1} \dfrac{119}{169}$ or 45(.2…)	M1dep	oe dep on 2nd M1
	$0.5 \times 6.5x \times 6.5x \times \sin$ their 45(.2…) or $15x^2$	M1dep	oe dep on 2nd and 3rd M1
	$21x^2$	A1	allow $p = 21$ if areas $6x^2$ and $15x^2$ seen
	Additional Guidance		
	Allow recovery of algebra eg1 $0.5 \times 4 \times 3 = 6$ is 1st M0 but if recovered to $6x^2$ scores 1st M1 eg2 Alt 1 $\sqrt{42.25 - 6.25} = 6$ is 2nd M0 and 3rd M0 but if recovered to $6x$ scores 2nd M1 and 3rd M1		
	Do not allow final mark if an incorrect area is seen eg do not allow answer $21x^2$ if their two areas are $6x^2$ and $15x$		
	Answer $21x^2$ with no incorrect working eg fully correct working with numbers and final answer $21x^2$		5 marks
	Allow recovery of missing brackets		
	Choose the scheme that favours the student		

Q	Answer	Mark	Comments
20(a)	**Alternative method 1**		
	Full method leading to angle *BCD* = 180 – 2x	M1	eg angle *CFE* = x and angle *FCE* = 180 – 2x and angle *BCD* = 180 – 2x
	Full reasoning for their method	A1	eg (base angles of) isosceles (triangle are equal) and (sum of) angles in a triangle (is 180) and (vertically) opposite angles
	angle *BAD* = 2x and (opposite angles of) cyclic quadrilateral (add to 180)	A1	must see M1
	Alternative method 2 Working out angle *DCF* using angle at centre		
	angle *DCF* = 2x	M1	
	angle at centre (is double angle at circumference)	A1	
	Full method leading to angle *BAD* = 2x and full reasoning for their method	A1	must see M1 eg angle *BCD* = 180 – 2x and angle *BAD* = 2x and angles on a (straight) line (add to 180) and (opposite angles of) cyclic quadrilateral (add to 180)

Mark scheme and Additional Guidance continue on the next three pages

Q	Answer	Mark	Comments
20(a) cont	**Alternative method 3 Working out angle *DCF* not using angle at centre**		
	Full method leading to angle *DCF* = 2x	M1	eg angle *CFE* = x and angle *DCF* = 2x
	Full reasoning for their method	A1	eg (base angles of) isosceles (triangle are equal) and exterior angle (of triangle is sum of interior opposite angles)
	Full method leading to angle *BAD* = 2x and full reasoning for their method	A1	must see M1 eg angle *BCD* = 180 – 2x and angle *BAD* = 2x and angles on a (straight) line (add to 180) and (opposite angles of) cyclic quadrilateral (add to 180)

Mark scheme and Additional Guidance continue on the next two pages

Q	Answer	Mark	Comments
20(a) cont	**Alternative method 4**		
	Full method leading to angle *DFC* = 90 – x and angle *ABC* = 90 – x	M1	eg angle *CFE* = x and angle *DFE* = 90 and angle *DFC* = 90 – x and angle *CDF* = 90 – x and angle *ADC* = 90 + x and angle *ABC* = 90 – x
	Full reasoning for their method	A1	eg (base angles of) isosceles (triangle are equal) and (angle in a) semicircle (is 90) and (sum of) angles in a triangle (is 180) and angles on a (straight) line (add to 180) and (opposite angles of) cyclic quadrilateral (add to 180)
	angle *BAD* = 2x and (sum of) angles in a triangle (is 180)	A1	must see M1

Additional Guidance is on the next page

Q	Answer	Mark	Comments

Q	Additional Guidance	
20(a) cont	It is possible to score M1A1A0 or M1A0A1	
	Do not award any marks from angles on the diagram	
	Angles must be stated unambiguously eg condone angle *B* but do not condone angle *D*	
	‘angle’ may be missing or replaced by a symbol - mark intention	
	angle *CFE* may be seen as angle *EFC* or angle *BFE* etc	
	For (base angles of) isosceles (triangle are equal) allow radii (are equal)	
	For (sum of) angles in a triangle (is 180) allow triangle is 180	
	Use judgement when considering wording of reasons and allow abbreviations	
	Alt 2 Final A1 reason may be exterior angle of cyclic quadrilateral (equals interior opposite angle)	
	Choose the scheme that favours the student	
	Ignore angles that are not needed for their scheme even if incorrect	
	Allow recovery of missing brackets	
	Starting with angle *BAD* = 2x	M0A0A0

Q	Answer	Mark	Comments
20(b)	30	B2	B1 correct equation or calculation eg $90 + 2x + x = 180$ or $90 - x = 2x$ or $3x = 90$ or $6x = 180$ or $90 \div 3$
	Additional Guidance		
	Ignore any expressions for angles and any other calculated angles		
	Ignore any reasons		

Q	Answer	Mark	Comments
21	$8^2 + 12^2$ or $64 + 144$ or 208 or $8^2 + 12^2 + 15^2$ or $64 + 144 + 225$ or 433	M1	HC^2 or CE^2 implied by 2nd M1
	$\sqrt{8^2+12^2}$ or $\sqrt{208}$ or $4\sqrt{13}$ or 14.4… or $\sqrt{8^2+12^2+15^2}$ or $\sqrt{433}$ or 20.8…	M1dep	oe may be on diagram fully correct trigonometry method leading to 14.4… or 20.8… can score M2 eg $8 \div \sin\left(\tan^{-1}\frac{8}{12}\right)$ or $8 \div \sin\left(\tan^{-1}\frac{8}{\sqrt{12^2+15^2}}\right)$
	$\tan x = \frac{15}{\sqrt{8^2+12^2}}$ or $\cos x = \frac{\sqrt{8^2+12^2}}{\sqrt{8^2+12^2+15^2}}$ or $\sin x = \frac{15}{\sqrt{8^2+12^2+15^2}}$	M1dep	oe eg $\tan x$ = [1.04, 1.042] or or $\cos x$ = [0.69, 0.6934] or $\sin x$ = [0.72, 0.7212] or $90 - \tan^{-1}\frac{\sqrt{8^2+12^2}}{15}$ dep on M2 any letter
	46(.1…)	A1	

Additional Guidance

3rd M1 If using sine rule or cosine rule, must be in the form $\cos x =$ or $\sin x =$ eg $\cos x = \frac{20.8^2+14.4^2-15^2}{2\times 20.8\times 14.4}$ (oe eg $\cos^{-1}$ [0.69, 0.6934])	
3rd M1 Condone $\tan = \frac{15}{\sqrt{8^2+12^2}}$ etc	M3
Allow the first 2 M marks even if not subsequently used	
Allow recovery of missing brackets	

Q	Answer		Mark	Comments
22(a)	**Alternative method 1**			
	$2\sin^2 x - 1 + 1 - \sin^2 x$ or $2\sin^2 x - (\sin^2 x + \cos^2 x) + \cos^2 x$ or $2\sin^2 x - \sin^2 x - \cos^2 x + \cos^2 x$ or $2\sin^2 x - \sin^2 x$ or $\sin^2 x - \cos^2 x + \cos^2 x$ or $1 + \sin^2 x - 1$		M1	use of $\sin^2 x + \cos^2 x = 1$ in numerator ignore any denominator
	$\dfrac{\sin^2 x}{\sin x \cos x}$ with M1 seen	$\dfrac{\sin^2 x}{\tan x \cos^2 x}$ with M1 seen	M1dep	simplification to one step from $\dfrac{\sin x}{\cos x}$ or simplification to one step from $\dfrac{\tan^2 x}{\tan x}$
	$\dfrac{\sin x}{\cos x}$ and $\tan x$ with M2 seen	$\dfrac{\tan^2 x}{\tan x}$ and $\tan x$ with M2 seen	A1	SC3 equates given expression to $\tan x$ and cross multiplies to show equivalence with full working shown

Mark scheme and Additional Guidance continue on the next two pages

Q	Answer		Mark	Comments
22(a) cont	**Alternative method 2**			
	$2(1 - \cos^2 x) - 1 + \cos^2 x$ or $2 - 2\cos^2 x - 1 + \cos^2 x$		M1	use of $\sin^2 x + \cos^2 x = 1$ in numerator ignore any denominator
	$\frac{1 - \cos^2 x}{\sin x \cos x}$ and $\frac{\sin^2 x}{\sin x \cos x}$ with M1 seen	$\frac{1 - \cos^2 x}{\sin x \cos x}$ and $\frac{\sin^2 x}{\tan x \cos^2 x}$ with M1 seen	M1dep	simplification to one step from $\frac{\sin x}{\cos x}$ or simplification to one step from $\frac{\tan^2 x}{\tan x}$
	$\frac{\sin x}{\cos x}$ and $\tan x$ with M2 seen	$\frac{\tan^2 x}{\tan x}$ and $\tan x$ with M2 seen	A1	SC3 equates given expression to $\tan x$ and cross multiplies to show equivalence with full working shown
	Alternative method 3			
	$\frac{2\sin x}{\cos x} - \frac{\sin^2 x}{\sin x \cos x}$		M1	from $\frac{2\sin^2 x}{\sin x \cos x} - \frac{1 - \cos^2 x}{\sin x \cos x}$
	$2\tan x - \frac{\sin^2 x}{\sin x \cos x}$ or $\frac{2\sin x}{\cos x} - \frac{\sin x}{\cos x}$ with M1 seen		M1dep	simplification to one step from $2\tan x - \tan x$
	$2\tan x - \tan x$ and $\tan x$ with M2 seen		A1	SC3 equates given expression to $\tan x$ and cross multiplies to show equivalence with full working shown

Additional Guidance is on the next page

Q	Answer	Mark	Comments

Q	Additional Guidance	
22(a) cont	Equating given expression to tan x and cross multiplying can score SC3 or M1M0A0 eg1 Alt 1 $\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = \tan x$ $2\sin^2 x - 1 + \cos^2 x = \tan x \sin x \cos x$ $2\sin^2 x - 1 + 1 - \sin^2 x = \tan x \sin x \cos x$ (scores M1 here for LHS)	M1M0A0
	eg2 $\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = \tan x$ $2\sin^2 x - 1 + \cos^2 x = \tan x \sin x \cos x$ $2\sin^2 x - 1 + 1 - \sin^2 x = \tan x \sin x \cos x$ $\sin^2 x = \tan x \sin x \cos x$ $\sin^2 x = \frac{\sin x}{\cos x} \sin x \cos x$ $\sin^2 x = \sin^2 x$	SC3
	Use of $\sin x = \frac{\text{opp}}{\text{hyp}}$ etc	M0M0A0
	Allow sin or s for sin x etc	
	Condone sin x^2 for $\sin^2 x$ etc	
	Allow any letter for x	
	Alts 1 and 2 For A1 $\frac{\sin x}{\cos x}$ is implied by $\frac{\sin^2 x}{\sin x \cos x}$ with cancelling shown	

Q	Answer	Mark	Comments
22(b)	135 and 315 with no other solutions [0, 360]	B2	B1 135 with no other solutions [0, 360] or 315 with no other solutions [0, 360] SC1 135 and 315 with one other solution [0, 360]
	Additional Guidance		
	Mark the answer line unless blank eg 135 and 315 in working with 135 on answer line		B1
	–45 and 135 and 315		B2
	–45 and 135		B1
	Ignore incorrect solutions outside the range [0, 360] eg 135 and 315 and –90		B2
	135 and 225 and 315		SC1
	Both answers embedded ie tan 135 tan 315		B1
	0 and 135 and 225 and 315		B0
	45 and 135		B0
	225 and 315		B0

Q	Answer	Mark	Comments
23(a)	(1, –3)	B1	
	Additional Guidance		
	Mark intention eg condone 1, –3		B1

Q	Answer	Mark	Comments
23(b)	**Alternative method 1**		
	$-3 + \sqrt{25}$ (= 2) or –3 + 5 (= 2)	B1	oe eg 5 – 3 (= 2) or 2 + 3 = 5
	Alternative method 2		
	$(y + 3)^2 = 25$ and $y = 2$ or $y + 3 = 5$ and $y = 2$ or $(2 + 3)^2 = 25$	B1	oe eg $(1 - 1)^2 + (y + 3)^2 = 25$ and $y = 2$
	Additional Guidance		
	(1, –3) + (0, 5) = (1, 2) so $y = 2$		B0
	Allow –3 + radius of 5		B1
	$2 = 0x + c$ $c = 2$ so $y = 2$		B0

Q	Answer	Mark	Comments
23(c)	**Alternative method 1 Using equation *PR***		
	$\frac{-7 - \text{their } -3}{4 - \text{their } 1}$ or $-\frac{4}{3}$	M1	oe grad *PC* their –3 and their 1 from (a)
	$-1 \div$ their $-\frac{4}{3}$ or $\frac{3}{4}$	M1	oe grad *PR* their $-\frac{4}{3}$ must be a value (gradient *PR* =) $\frac{3}{4}$ is M2
	$2 - -7 =$ their $\frac{3}{4}(x - 4)$	M1dep	oe equation *PR* with $y = 2$ substituted eg $2 = \frac{3}{4}x - 10$ dep on 2nd M1
	16	A1ft	only ft their –3 and their 1 from (a)
	Alternative method 2 Using $RC^2 = CP^2 + PR^2$ or $PR^2 = QR^2$ with *R* (x, 2)		
	$(x - \text{their } 1)^2 + (2 - \text{their } -3)^2$ $= (2 - \text{their } -3)^2 + (x - 4)^2 + (2 - -7)^2$	M1	oe eg $(x - 1)^2 = (x - 4)^2 + (2 - -7)^2$ their –3 and their 1 from (a)
	$x^2 - 2x + 1 + 25$ $= 25 + x^2 - 8x + 16 + 81$	M1dep	oe brackets expanded
	$96 = 6x$ or $96 \div 6$	M1dep	oe linear equation or calculation dep on M2
	16	A1ft	only ft their –3 and their 1 from (a)

Mark scheme and Additional Guidance continue on the next three pages

Q	Answer	Mark	Comments
23(c) cont	**Alternative method 3 Using equation *CR***		
	$\dfrac{-7-2}{4-\text{their } 1}$ or –3	M1	oe grad *PQ* their 1 from (a)
	–1 ÷ their –3 or $\dfrac{1}{3}$	M1	oe grad *CR* their –3 must be a value (gradient *CR* =) $\dfrac{1}{3}$ is M2
	2 – their –3 = their $\dfrac{1}{3}$ (x – their 1)	M1dep	oe equation *CR* with y = 2 substituted eg $2 = \dfrac{1}{3}x - \dfrac{10}{3}$ dep on 2nd M1
	16	A1ft	only ft their –3 and their 1 from (a)
	Alternative method 4 Using equation *MR* where *M* is the midpoint of *PQ*		
	$\dfrac{-7-2}{4-\text{their } 1}$ or –3	M1	oe grad *PQ* their 1 from (a)
	–1 ÷ their –3 or $\dfrac{1}{3}$	M1	oe grad *MR* their –3 must be a value (gradient *MR* =) $\dfrac{1}{3}$ is M2
	$\left(\dfrac{4+\text{their } 1}{2}, \dfrac{-7+2}{2}\right)$ or (2.5, –2.5) and 2 – their –2.5 = their $\dfrac{1}{3}$ (x – their 2.5)	M1dep	oe midpoint of *PQ* and equation *MR* with y = 2 substituted eg $2 = \dfrac{1}{3}x - \dfrac{10}{3}$ dep on 2nd M1
	16	A1ft	only ft their 1 from (a)

Mark scheme and Additional Guidance continue on the next two pages

Q	Answer	Mark	Comments
23(c) cont	**Alternative method 5 Using equation *MC* where *M* is the midpoint of *PQ***		
	$\left(\frac{4 + \text{their } 1}{2}, \frac{-7+2}{2}\right)$ or (2.5, –2.5)	M1	oe midpoint of *PQ* their 1 from (a)
	$\frac{\text{their } -3 - \text{their } -2.5}{\text{their } 1 - \text{their } 2.5}$ or $\frac{1}{3}$	M1dep	oe grad *MC*
	2 – their –3 = their $\frac{1}{3}$ (x – their 1) or 2 – their –2.5 = their $\frac{1}{3}$ (x – their 2.5)	M1dep	oe equation *MC* with y = 2 substituted eg $2 = \frac{1}{3}x - \frac{10}{3}$ dep on M2
	16	A1ft	only ft their –3 and their 1 from (a)
	Alternative method 6 Using trigonometry where *M* is the midpoint of *PQ*		
	(*QM* =) $\frac{1}{2}\sqrt{(4 - \text{their } 1)^2 + (-7-2)^2}$ or $\frac{1}{2}\sqrt{90}$ or 4.74…	M1	
	$\sin^{-1}\left(\frac{\text{their } 4.74...}{5}\right)$ or (angle *QCM* =) 71.5… or 71.6	M1dep	oe angle *QCM*
	tan (their 71.5…) = $\frac{x - \text{their } 1}{5}$	M1dep	using triangle *QCR*
	16	A1ft	only ft their 1 from (a)

Additional Guidance is on the next page

Q	Answer	Mark	Comments
23(c) cont	**Additional Guidance**		
	Allow (16, …) to imply answer 16		
	Alt 1 $-\frac{4}{3}x$ is M0 unless recovered		
	(a) (1, –2) grad *PC* = $-\frac{5}{3}$ grad *PR* = $\frac{3}{5}$ Answer 19 (3rd M1 can be implied by A1ft answer)	M1M1 M1A1ft	

Q	Answer	Mark	Comments
24	$3x^4$ or $4x^3$	M1	oe eg $5 \times \frac{3}{5}x^{5-1}$
	$3x^4 + 4x^3$	A1	
	$x^3(3x + 4)$ (= 0)	M1dep	allow partial factorisation of their $3x^4 + 4x^3$ if at least x is taken as a factor ft their two terms if M1 scored
	$x^3(3x + 4)$ (= 0) and (x =) 0 and (x =) $-\frac{4}{3}$ with no other solutions	A1	allow partial factorisation if at least x is taken as a factor
	Additional Guidance		
	$3x^4 + 4x^3 = 0$ $x = 0$ and $x = -\frac{4}{3}$	M1A1 M0A0	
	Condone $y = 3x^4 + 4x^3$	M1A1	
	Ignore higher derivatives		
	Condone (0, …) and $\left(-\frac{4}{3}, ...\right)$ for (x =) 0 and (x =) $-\frac{4}{3}$		
	Allow –1.33…for $-\frac{4}{3}$ (ignore any incorrect conversion attempt after $-\frac{4}{3}$ seen)		

Q	Answer	Mark	Comments
25	**Alternative method 1**		
	$(-c)^3 - 10(-c) - c$ (= 0) or $-c^3 + 10c - c$ (= 0) or $-c^3 + 9c$ (= 0)	M1	oe
	$c(9 - c^2)$ (= 0) or $c(3 + c)(3 - c)$ (= 0) or $c^2 = 9$	M1dep	oe factorised expression or quadratic equation
	3 with no other value(s)	A1	SC2 answer 3 with one or both of –3 and 0 and no other value
	Alternative method 2		
	$(x + c)(x^2 - cx - 1)$	M1	
	$-1 - c^2 = -10$	M1dep	oe quadratic equation
	3 with no other value(s)	A1	SC2 answer 3 with one or both of –3 and 0 and no other value

Additional Guidance

$(-3)^3 - 10(-3) - 3 = 0$ and Answer 3 (no part marks)	M2A1
$(-3)^3 - 10(-3) - -3 = 0$ and Answer 3	Zero
$3^3 - 10(3) - -3 = 0$ and Answer 3	Zero
Answer 3 with no incorrect working	M2A1
Allow recovery of missing brackets	

Q	Answer	Mark	Comments
26	**Alternative method 1**		
	$(x + 3)^2$….	M1	
	$(x + 3)^2 - 3^2 - a$ or $(x + 3)^2 - 3^2 \geqslant a$ or $(x + 3)^2 \geqslant a + 3^2$	M1dep	oe expression or inequality eg $(x + 3)^2 \geqslant 9 + a$ allow $\geqslant$ to be any inequality symbol or = eg allow $(x + 3)^2 - 9 = a$ implies M2
	$-3^2 - a \geqslant 0$ or $-3^2 - a > 0$	M1dep	oe inequality eg $-9 - a \geqslant 0$ or $-9 - a > 0$ or $a < -9$ implies M3
	$a \leqslant -9$ or $-9 \geqslant a$	A1	SC1 $x^2 + 6x - a \geqslant 0$ oe inequality (may be seen in working lines)
	Alternative method 2		
	$2x + 6 = 0$	M1	must have = 0
	(minimum at) $x = -3$	M1dep	implies M2 $x = -3$ must be the only value or be clearly chosen
	$(-3)^2 + 6 \times (-3) - a \geqslant 0$ or $(-3)^2 + 6 \times (-3) - a > 0$	M1dep	oe inequality eg $9 - 18 - a \geqslant 0$ or $9 - 18 - a > 0$ or $a < -9$ implies M3
	$a \leqslant -9$ or $-9 \geqslant a$	A1	SC1 $x^2 + 6x - a \geqslant 0$ oe inequality (may be seen in working lines)

Mark scheme and Additional Guidance continue on the next page

Q	Answer	Mark	Comments
26 cont	**Alternative method 3**		
	$6^2 - 4 \times 1 \times -a$	M1	$b^2 - 4ac$ must be selected if seen in quadratic formula
	$6^2 - 4 \times 1 \times -a \leqslant 0$ or $6^2 - 4 \times 1 \times -a < 0$	M1dep	oe inequality implies M2
	$36 + 4a \leqslant 0$ or $36 + 4a < 0$	M1dep	oe inequality eg $4a \leqslant -36$ implies M3
	$a \leqslant -9$ or $-9 \geqslant a$	A1	SC1 $x^2 + 6x - a \geqslant 0$ oe inequality (may be seen in working lines)
	Additional Guidance		
	Alt 1 2nd M1 Any inequality symbol or = allowed 3rd M1 Only the inequality symbols shown are allowed (do not allow =)		
	Allow $(x + 3)(x + 3)$ for $(x + 3)^2$		

Q	Answer	Mark	Comments
27	**Alternative method 1**		
	Shows substitution of a value of $x < -2$ into $\frac{dy}{dx}$ and shows substitution of a value of $x > -2$ into $\frac{dy}{dx}$	M1	eg $(-3 + 2)^6 + (-3 + 2)^4$ and $(-1 + 2)^6 + (-1 + 2)^4$ allow $(-1)^6 + (-1)^4$ with $x = -3$ stated and $(1)^6 + (1)^4$ with $x = -1$ stated
	Evaluates both correctly or states that each is positive with M1 seen	M1dep	eg $(-3 + 2)^6 + (-3 + 2)^4 = 2$ and $(-1 + 2)^6 + (-1 + 2)^4 = 2$ allow $(-1)^6 + (-1)^4 = 2$ with $x = -3$ stated and $(1)^6 + (1)^4 = 2$ with $x = -1$ stated
	Statement with M2 seen	A1	eg either side of P $\frac{dy}{dx} > 0$ with M2 seen SC2 states two values of x (one < -2 and one > -2) and shows the correct value of $\frac{dy}{dx}$ for each and makes a statement SC1 states two values of x (one < -2 and one > -2) and shows the correct value of $\frac{dy}{dx}$ for each

Mark scheme and Additional Guidance continue on the next three pages

Q	Answer	Mark	Comments
27 cont	**Alternative method 2**		
	$x < -2$ and $(-)^6 + (-)^4$ and $x > -2$ and $(+)^6 + (+)^4$	M1	allow without brackets allow less than for $<$ etc
	$x < -2$ and $(-)^6 + (-)^4 > 0$ and $x > -2$ and $(+)^6 + (+)^4 > 0$	M1dep	allow without brackets allow $= +$ for > 0
	Statement with M2 seen	A1	eg either side of P $\frac{dy}{dx} > 0$ with M2 seen SC2 states two values of x (one < –2 and one > –2) and shows the correct value of $\frac{dy}{dx}$ for each and makes a statement SC1 states two values of x (one < –2 and one > –2) and shows the correct value of $\frac{dy}{dx}$ for each

Additional Guidance is on the next two pages

Q	Answer	Mark	Comments

Additional Guidance

For A1 a clear statement is needed after M2 scored

Examples of acceptable statements with M2 seen

eg1 For $x < -2$ gradient is + and for $x > -2$ gradient is +

eg2 To the left of P $m > 0$

To the right of P $m > 0$

eg3 (When both of their substitutions correctly evaluate to the same value)

They are the same **positive** value

eg4 **Both** gradients are the same sign

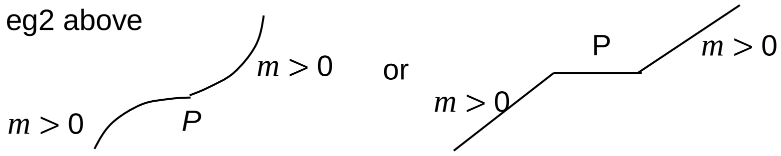
eg5 m is + **both** times

eg6 Gradient is **always** positive (apart from at P)

eg7 **Function (or curve)** is increasing (either side of P)

Allow a statement to be made using a diagram with M2 seen

eg accept for eg2 above



Allow a statement to be made using a table with M2 seen

eg accept for eg1 above

–3	–2	–1
+	0	+

When both of their substitutions correctly evaluate to the same positive value condone for the statement with M2 seen

Gradients are the same (implies both positive)

Do not accept for the statement

eg1 Gradient is increasing

eg2 m is positive

eg3 Gradient is positive

eg4 P is a point of inflection

Additional Guidance continues on the next page

Q	Answer	Mark	Comments

Q	Additional Guidance	
27 cont	Allow gradient or m for $\frac{dy}{dx}$	
	For evaluations allow rounding or truncating to 1 sf or better	
	Ignore higher derivatives	
	Ignore substitution of $x = –2$	
	$x = –3$ gradient = 2 $x = –1$ gradient = 2 either side of P gradient > 0	SC2
	<table><tr><td>–3</td><td>–2</td><td>–1</td></tr><tr><td>2</td><td>0</td><td>2</td></tr></table> gradient is positive both times	SC2
	<table><tr><td>–3</td><td>–2</td><td>–1</td></tr><tr><td>2</td><td>0</td><td>2</td></tr></table>	SC1