Edexcel GCSE
Mathematics (Linear) – 1MA0

PROOF

Materials required for examination	**Items included with question papers**
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser. Tracing paper may be used.	Nil

Instructions

Use black ink or ball-point pen.
Fill in the boxes at the top of this page with your name, centre number and candidate number.
Answer all questions.
Answer the questions in the spaces provided – there may be more space than you need.
Calculators may be used.

Information

The marks for each question are shown in brackets – use this as a guide as to how much time to spend on **each** question.
Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed – you should take particular care on these questions with your spelling, punctuation and grammar, as well as the clarity of expression.

Advice

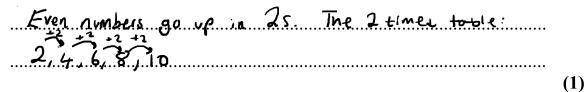Read each question carefully before you start to answer it.
Keep an eye on the time.
Try to answer every question.
Check your answers if you have time at the end.

1. The *n*th even number is 2*n*.

   The next even number after 2*n* is 2*n* + 2

   (a) Explain why.

   Even numbers go up in 2s. The 2 times table:
   2, 4, 6, 8, 10

   (1)

   (b) Write down an expression, in terms of *n*, for the next even number after 2*n* + 2

   $2n + 4$

   (1)

   (c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6

   $2n + (2n+2) + (2n+4)$

   $6n + 6$

   $6(n+1)$

   Because we can factorise out 6, $6n+6$ is always a multiple of 6.

   (3)
   (5 marks)

2. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n.

$$(3n+1)(3n+1) \; - \; (3n-1)(3n-1)$$

$$(9n^2+3n+3n+1) \; - \; (9n^2-3n-3n+1)$$

$$(9n^2+6n+1) \; - \; (9n^2-6n+1)$$

$$9n^2+6n+1 \; -9n^2+6n-1$$

$$12n$$

$$\underline{\underline{4(3n)}}$$

(3 marks)

3. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

$$n + n+1$$

$$2n+1$$

$2n$ is an even number. An even number plus one is always odd.

4. Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of 8}$$

for all positive integer values of n.

$$(2n+3)(2n+3) - (2n-3)(2n-3)$$

$$(4n^2 + 6n + 6n + 9) - (4n^2 - 6n - 6n + 9)$$

$$(4n^2 + 12n + 9) - (4n^2 - 12n + 9)$$

$$24n$$

$$\underline{8(3n)}$$

(3 marks)

*5. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

n and $n+1$ (two consecutive integers)

$(n+1)^2 - n^2$ (diff. between squares)

$(n+1)(n+1) - n^2$

$n^2 + n + n + 1 - n^2$

$\underline{2n + 1}$

Sum of n and $n+1$ $= \underline{2n+1}$

(4 marks)

6. Prove that $(5n + 1)^2 - (5n - 1)^2$ is a multiple of 5, for all positive integer values of n.

$$(5n+1)(5n+1) \quad - \quad (5n-1)(5n-1)$$

$$(25n^2 + 5n + 5n + 1) - (25n^2 - 5n - 5n + 1)$$

$$(25n^2 + 10n + 1) - (25n^2 - 10n + 1)$$

$$20n$$

$$\underline{5(4n)}$$

7. If *2n* is always even for all positive integer values of *n*, prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

$$2n \quad \text{and} \quad 2n+2$$

$$(2n)^2 + (2n+2)^2$$

$$4n^2 + (2n+2)(2n+2)$$

$$4n^2 + 4n^2 + 4n + 4n + 4$$

$$8n^2 + 8n + 4$$

$$4(2n^2 + 2n + 1)$$

8. Prove that

$(n + 1)^2 - (n - 1)^2 + 1$ is always odd for all positive integer values of n.

$(n+1)(n+1) - (n-1)(n-1) + 1$

$(n^2+n+n+1) - (n^2-n-n+1) +1$

$(n^2+2n+1) - (n^2-2n+1) +1$

$n^2+2n+1 - n^2+2n-1 +1$

$4n+1$

The four times table is always even. An even plus one is odd.

(3 marks)

9. Prove algebraically that the sum of the squares of any two consecutive numbers always leaves a remainder of 1 when divided by 4.

$$n \quad \text{and} \quad n+1$$

$$n^2 + (n+1)^2$$

$$n^2 + (n+1)(n+1)$$

$$n^2 + n^2 + n + n + 1$$

$$2n^2 + 2n + 1$$

$$2n(n+1) + 1$$

$$2(n)(n+1) + 1$$

$2 \times$ an even is always a multiple of 4, either n or $n+1$ is even. $\therefore 2(n)(n+1)$ is divisible by 4. There is a

remainder of 1.

(4 marks)